

# Model Predictive Control Toolbox

For Use with MATLAB®

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- Computation
- Visualization
- Programming

Getting Started

*Version 2*



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### *Getting Started with the Model Predictive Control Toolbox*

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## Introduction

1

<b>What Is the Model Predictive Control Toolbox? . . . . .</b>	<b>1-2</b>
<b>Using the Documentation . . . . .</b>	<b>1-4</b>
Related Products . . . . .	1-5
<b>Bibliography . . . . .</b>	<b>1-6</b>

## Building Models

2

<b>Overview . . . . .</b>	<b>2-2</b>
Plant Model . . . . .	2-2
Plant Inputs and Outputs . . . . .	2-3
<b>Linear, Time Invariant (LTI) Models . . . . .</b>	<b>2-4</b>
Transfer Function Format . . . . .	2-4
Zero/Pole/Gain Format . . . . .	2-5
State-Space Format . . . . .	2-5
LTI Object Properties . . . . .	2-7
Multiinput Multioutput (MIMO) Plants . . . . .	2-11
LTI Model Characteristics . . . . .	2-13
<b>System Identification Toolbox Models . . . . .</b>	<b>2-14</b>
System Identification Model Definition Example . . . . .	2-14
Converting a System Identification Toolbox Model to an LTI Object . . . . .	2-15
Step-Response Models . . . . .	2-17
<b>Using Simulink to Develop LTI Models . . . . .</b>	<b>2-19</b>
Linearization Using Simulink Control Design . . . . .	2-19
Linearization Using Simulink Functions . . . . .	2-24

<b>Bibliography</b> .....	<b>2-27</b>
---------------------------	-------------

## **The Design Tool**

### **3**

<b>Introduction</b> .....	<b>3-2</b>
Starting the Design Tool .....	<b>3-2</b>
Loading a Plant Model .....	<b>3-3</b>
Navigation Using the Tree View .....	<b>3-7</b>
<b>Linear Simulations</b> .....	<b>3-12</b>
Defining Simulation Conditions .....	<b>3-12</b>
Running a Simulation .....	<b>3-14</b>
Open-Loop Simulations .....	<b>3-16</b>
<b>Changing Controller Settings</b> .....	<b>3-20</b>
Model and Horizons .....	<b>3-20</b>
Weight Tuning .....	<b>3-22</b>
Blocking .....	<b>3-27</b>
Defining Manipulated Variable Constraints .....	<b>3-30</b>
Disturbance Modeling and Estimation .....	<b>3-34</b>
<b>Defining Soft Output Constraints</b> .....	<b>3-42</b>
<b>Robustness Testing</b> .....	<b>3-46</b>
Plant Model Perturbation .....	<b>3-46</b>
Simulation Tests .....	<b>3-46</b>
<b>Plant Models with Delays</b> .....	<b>3-49</b>
Importing the Plant Model .....	<b>3-49</b>
Specifying Controller Horizons .....	<b>3-50</b>
<b>Nonsquare Plants</b> .....	<b>3-55</b>
More Outputs Than Manipulated Variables .....	<b>3-55</b>
More Manipulated Variables Than Outputs .....	<b>3-56</b>
<b>Nonlinear Plants</b> .....	<b>3-57</b>

The MPC Controller Block .....	3-57
Initiating the Controller Design .....	3-58
Validating the Linearized Model .....	3-61
Modifying the Linearized Model .....	3-63
Linear Simulation Tests .....	3-64
Nonlinear Simulation Tests .....	3-66
Modifying the Controller Using the Design Tool .....	3-68
Exiting the Design Tool .....	3-68
<b>Saving Your Work</b> .....	<b>3-69</b>
Exporting a Controller .....	3-69
Saving a Project .....	3-70
<b>Loading Your Saved Work</b> .....	<b>3-72</b>

## Using Model Predictive Control Toolbox Functions

# 4

<b>Controller Definition</b> .....	<b>4-2</b>
Creating a Controller Object .....	4-2
Viewing and Altering Controller Properties .....	4-3
<b>Linear Simulations</b> .....	<b>4-6</b>
Using the sim Function .....	4-6
Saving Calculated Results .....	4-6
Simulation Options .....	4-7
<b>Simulations Involving Nonlinear Plants</b> .....	<b>4-8</b>
A Nonlinear CSTR Application .....	4-8
Example Code for Successive Linearization .....	4-9
CSTR Results and Discussion .....	4-10
<b>Analysis Tools</b> .....	<b>4-14</b>
Steady-State Gain Computation .....	4-14
Controller Extraction .....	4-14
<b>Bibliography</b> .....	<b>4-16</b>



# Introduction

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What Is the Model Predictive Control Toolbox? (p. 1-2)

Using the Documentation (p. 1-4)

Bibliography (p. 1-6)

Compares the Model Predictive Control Toolbox to other common system control approaches

Describes the available documentation

Suggestions for further reading

## What Is the Model Predictive Control Toolbox?

The Model Predictive Control Toolbox is a collection of software that helps you design, analyze, and implement an advanced industrial automation algorithm. Like other MATLAB® tools, it provides a convenient graphical user interface (GUI) as well as a flexible command syntax that supports customization.

A Model Predictive Control Toolbox controller automates a target system (the *plant*) by combining a prediction and a control strategy. An approximate plant model provides the prediction. The control strategy compares predicted plant signals to a set of objectives, then adjusts available actuators to achieve the objectives while respecting the plant's constraints.

The controller's constraint-tolerance differentiates it from other optimal control strategies (e.g., the Linear-Quadratic-Gaussian approach supported in the Control System Toolbox). The impetus for this is industrial experience suggesting that the drive for profitability often pushes the plant to one or more constraints. The Model Predictive Control Toolbox controller considers such factors explicitly, allowing it to allocate the available plant resources intelligently as the system evolves over time.

The Model Predictive Control Toolbox uses the same powerful linear dynamic modeling tools found in the Control System Toolbox and System Identification Toolbox. You can employ transfer functions, state-space matrices, or a combination. You can also include delays, which are a common feature of industrial plants.

If you don't have a model but can perform experiments, the System Identification Toolbox provides methods to help you develop a data-based model for use in the Model Predictive Control Toolbox.

If you'd rather use the Simulink® graphical tools to model your plant, the Model Predictive Control Toolbox provides a special controller block for that environment. For example, you can linearize a nonlinear Simulink model, use the linearized model to build a Model Predictive Control Toolbox controller, and evaluate its ability to control the nonlinear model. Once that is working well, you can implement the control strategy in a real plant using Real Time Workshop®.

For a list of books on predictive control theory and practice, see "Bibliography" on page 1-6. In particular, Maciejowski [4] illustrates and extends Version 1.0



of the Model Predictive Control Toolbox. (The command format used in [4] is obsolete in Model Predictive Control Toolbox Version 2.0, however.)

## Using the Documentation

**If you have limited experience with MATLAB or Model Predictive Control**, read this guide first. It shows how to

- Define your plant using the Control System Toolbox modeling tools (LTI transfer function and state space models)
- Derive a linear plant model from a nonlinear Simulink representation
- Design Model Predictive Control for your plant using `mpctool`, the graphical user interface (GUI), or special commands
- Simulate Model Predictive Control performance using `mpctool`, Simulink, or commands

**If you have experience with an earlier release of the Model Predictive Control Toolbox**, we advise you to read this book to familiarize yourself with the many new features and the new command syntax (the earlier syntax is still available, but the underlying code is no longer supported).

**If you need more details**, see the online documentation. To access it type

```
helpdesk
```

at the MATLAB prompt. When the help dialog window appears, select **Model Predictive Control Toolbox** in the **Contents** pane. This displays a roadmap with links to the available documentation components. Briefly, these are

- “Getting Started.” The online version of this manual.
- “MPC Problem Setup.” Mathematical details of the Model Predictive Control Toolbox algorithm and user specifications required for controller design.
- “MPC Simulink Library.” Describes the Model Predictive Control Controller Block and its use within Simulink.
- “Case-Study Examples.” Example Toolbox applications.
- “The Design Tool.” Reference manual for `mpctool`, the GUI.
- “Functions.” Reference manual describing each Model Predictive Control Toolbox function (used for controller design and simulation in MATLAB commands and scripts).
- “Blocks.” Reference manual describing the Model Predictive Control Toolbox blocks (used for controller design and simulation in Simulink).

- “Object Reference.” Details of the controller object that represents a complete controller design.
- “Release Notes.” Summarizes major features of this release, known limitations, etc.

## **Related Products**

The MathWorks provides other products that complement and enhance Model Predictive Control Toolbox functionality. See <http://www.mathworks.com/products/mpc/> for more information.

## **Bibliography**

- [1] Allgower, F., and A. Zheng, *Nonlinear Model Predictive Control*, Springer-Verlag, 2000.
- [2] Camacho, E. F., and C. Bordons, *Model Predictive Control*, Springer-Verlag, 1999.
- [3] Kouvaritakis, B., and M. Cannon, *Non-Linear Predictive Control: Theory & Practice*, IEE Publishing, 2001.
- [4] Maciejowski, J. M., *Predictive Control with Constraints*, Pearson Education POD, 2002.
- [5] Prett, D., and C. Garcia, *Fundamental Process Control*, Butterworths, 1988.
- [6] Rossiter, J. A., *Model-Based Predictive Control: A Practical Approach*, CRC Press, 2003.

# Building Models

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Overview (p. 2-2)

Describes the model and input/output signal conventions used in the Model Predictive Control Toolbox

Linear, Time Invariant (LTI) Models (p. 2-4)

Defines transfer function, zero-pole, and state-space models for use in the Model Predictive Control Toolbox

System Identification Toolbox Models (p. 2-14)

Imports System Identification Toolbox (data-based) models for use in the Model Predictive Control Toolbox.

Using Simulink to Develop LTI Models (p. 2-19)

Obtains a linear approximation of a nonlinear Simulink model.

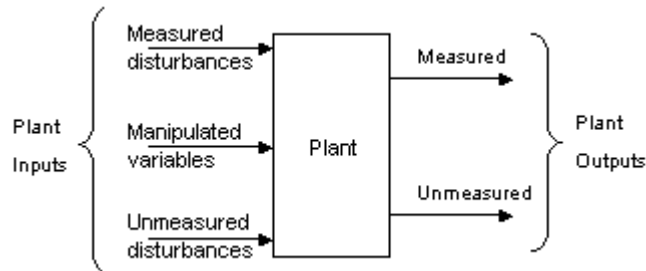
## Overview

This section covers the following topics:

- “Plant Model” on page 2-2
- “Plant Inputs and Outputs” on page 2-3

### Plant Model

The *plant* is the system (process or device) you intend to control. Figure 2-1 shows a schematic example.



**Figure 2-1: Plant with Input and Output Signals**

A Model Predictive Control Toolbox design requires a *plant model*, which defines the mathematical relationship between the plant inputs and outputs. The controller uses it to predict plant behavior.

The Model Predictive Control Toolbox requires the model to be linear, time invariant (LTI). You can define such a model as follows:

- Create a transfer function, state space, or zero/pole/gain model using methods provided by the Control System Toolbox
- Derive it from plant data using, e.g., methods provided by the System Identification Toolbox
- Derive it by linearizing a Simulink model

This chapter illustrates each of these approaches. The Control System Toolbox, Simulink, and the System Identification Toolbox documentation provides additional examples and details.

---

## Plant Inputs and Outputs

### Inputs

The *plant inputs* are the independent variables affecting the plant. As shown in Figure 2-1, there are three types:

**Measured disturbances.** The controller can't adjust them, but uses them for feedforward compensation.

**Manipulated variables.** The controller adjusts these in order to achieve its goals.

**Unmeasured disturbances.** These are independent inputs of which the controller has no direct knowledge, and for which it must compensate

### Outputs

The *plant outputs* are the dependent variables (outcomes) you wish to control or monitor. As shown in Figure 2-1, there are two types:

**Measured outputs.** The controller uses these to estimate unmeasured quantities and as feedback on the success of its adjustments.

**Unmeasured outputs.** The controller estimates these based on available measurements and the plant model. The controller can also hold unmeasured outputs at setpoints or within constraint boundaries.

You must specify the input and output types when designing the controller. See “Input and Output Types” on page 2-9 for more details.

## Linear, Time Invariant (LTI) Models

The Model Predictive Control Toolbox supports the same LTI model formats as the Control System Toolbox. You can use whichever is most convenient for your application. It's also easy to convert from one format to another.

The following sections describe the three model formats and the commands used to construct them:

- “Transfer Function Format” on page 2-4
- “Zero/Pole/Gain Format” on page 2-5
- “State-Space Format” on page 2-5
- “LTI Object Properties” on page 2-7
- “Multiinput Multioutput (MIMO) Plants” on page 2-11
- “LTI Model Characteristics” on page 2-13

For more details, see the Control System Toolbox documentation.

### Transfer Function Format

A transfer function (TF) relates a particular input/output pair. For example, if  $u(t)$  is a plant input and  $y(t)$  is an output, the transfer function relating them might be

$$\frac{Y(s)}{U(s)} = G(s) = \frac{s + 2}{s^2 + s + 10} e^{-1.5s}$$

This TF consists of a *numerator* polynomial,  $s + 2$ , a *denominator* polynomial,  $s^2 + s + 10$ , and a delay, which is 1.5 time units here. You can define  $G$  using the Control System Toolbox `tf` function:

```
Gtf1 = tf([1 2], [1 1 10], 'OutputDelay', 1.5)
```

The Control System Toolbox builds and displays it as follows:

```
Transfer function:
              s + 2
exp(-1.5*s) * -----
              s^2 + s + 10
```



## Zero/Pole/Gain Format

Like the TF, the zero/pole/gain (ZPK) format relates an input/output pair. The difference is that the ZPK numerator and denominator polynomials are factored, as in

$$G(s) = 2.5 \frac{s + 0.45}{(s + 0.3)(s + 0.1 + 0.7i)(s + 0.1 - 0.7i)}$$

(zeros and/or poles are complex numbers in general).

You define the ZPK model by specifying the zero(s), pole(s), and gain as in

$$\text{Gzpk1} = \text{zpk}(-0.45, [-0.3, -0.1+0.7*i, -0.1-0.7*i], 2.5)$$

## State-Space Format

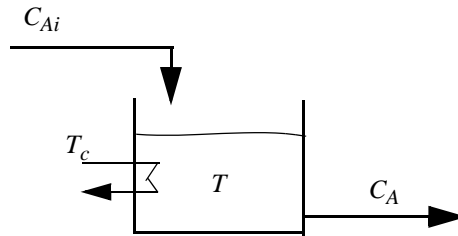
### Chemical Reactor Example

The state-space format is convenient if your model is a set of LTI differential and algebraic equations. For example, consider the following linearized model of a continuous stirred-tank reactor (CSTR) involving an exothermic (heat-generating) reaction [9]

$$\frac{dC'_A}{dt} = a_{11}C'_A + a_{12}T^n + b_{11}T'_c + b_{12}C'_{Ai}$$

$$\frac{dT^n}{dt} = a_{21}C'_A + a_{22}T^n + b_{21}T'_c + b_{22}C'_{Ai}$$

where  $C'_A$  is the concentration of a key reactant,  $T^n$  is the temperature in the reactor,  $T'_c$  is the coolant temperature,  $C'_{Ai}$  is the reactant concentration in the reactor feed, and  $a_{ij}$  and  $b_{ij}$  are constants. See the process schematic in Figure 2-2. The primes (e.g.,  $C'_A$ ) denote a deviation from the nominal steady-state condition at which the model has been linearized.



**Figure 2-2: CSTR Schematic**

Measurement of reactant concentrations is often difficult, if not impossible. Let us assume that  $T$  is a measured output,  $C_A$  is an unmeasured output,  $T_c$  is a manipulated variable, and  $C_{Ai}$  is an unmeasured disturbance.

### State-Space Format

The model fits the general state-space format

$$\frac{dx}{dt} = Ax + Bu$$

$$y = Cx + Du$$

where

$$x = \begin{bmatrix} C_A \\ T \end{bmatrix}, u = \begin{bmatrix} T_c \\ C_{Ai} \end{bmatrix}, y = \begin{bmatrix} T \\ C_A \end{bmatrix},$$

$$A = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}, B = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}, C = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}, D = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

The following code shows how to define such a model for some specific values of the  $a_{ij}$  and  $b_{ij}$  constants:

```
A = [-0.0285  -0.0014
      -0.0371  -0.1476];
B = [-0.0850   0.0238
      0.0802   0.4462];
```

```

C = [0 1
     1 0];
D = zeros(2,2);
CSTR = ss(A,B,C,D);

```

This defines a *continuous-time* state space model (no sampling period has been specified, so it defaults to zero). You can also specify discrete-time state space models and add delays to either type. See the Control System Toolbox documentation for details.

## LTI Object Properties

The `ss` function in the last line of the above code creates a state space model, `CSTR`, which is an *LTI object*. The `tf` and `zpk` commands described in sections “Transfer Function Format” on page 2-4 and “Zero/Pole/Gain Format” on page 2-5 also create LTI objects. Such objects contain the model parameters as well as optional properties.

### LTI Properties for the CSTR Example

The following code sets some of the `CSTR` model’s optional properties:

```

CSTR.InputName = {'T_c', 'C_A_i'};
CSTR.OutputName = {'T', 'C_A'};
CSTR.StateName = {'C_A', 'T'};
CSTR.InputGroup.MV = 1;
CSTR.InputGroup.UD = 2;
CSTR.OutputGroup.MO = 1;
CSTR.OutputGroup.UO = 2;
CSTR

```

The first three lines specify labels for the input, output and state variables. The next four specify the signal type for each input and output. The designations MV, UD, MO, and UO mean *manipulated variable*, *unmeasured disturbance*, *measured output*, and *unmeasured output*. (See “Plant Inputs and Outputs” on page 2-3 for definitions of these terms.) For example, the code specifies that input 2 of model `CSTR` is an unmeasured disturbance. The last line causes the LTI object to be displayed, generating the following lines in the MATLAB Command Window:

```
a =
      C_A      T
C_A -0.0285 -0.0014
T   -0.0371 -0.1476
```

```
b =
      T_c      C_Ai
C_A -0.085  0.0238
T   0.0802  0.4462
```

```
c =
      C_A      T
T      0      1
C_A    1      0
```

```
d =
      T_c      C_Ai
T      0      0
C_A    0      0
```

Input groups:

Name	Channels
MV	1
UD	2

Output groups:

Name	Channels
MO	1
UO	2

Continuous-time model

### Input and Output Names

The optional `InputName` and `OutputName` properties affect the model displays, as in the above example. The Model Predictive Control Toolbox also uses the `InputName` and `OutputName` properties to label plots and tables. In that context, the underscore character causes the next character to be displayed as a subscript.

## Input and Output Types

**General Case.** As mentioned in “Overview” on page 2-2, the Model Predictive Control Toolbox supports three input types and two output types. In an Model Predictive Control Toolbox design, designation of the input and output types determines the controller dimensions and has other important consequences.

For example, suppose your plant structure were as follows:

Plant Inputs	Plant Outputs
Two manipulated variables (MV)	Three measured outputs (MO)
One measured disturbance (MD)	Two unmeasured outputs (UO)
Two unmeasured disturbances (UD)	

The resulting controller would have four inputs (the three MOs and the MD) and two outputs (the MVs). It would include feedforward compensation for the measured disturbance, and would assume that you wanted the unmeasured disturbances and outputs to be included as part of the regulator design.

If you didn't want a particular signal to be treated as one of the above types, you could do one of the following:

- Eliminate the signal before using the model in controller design.
- For an output, designate it as unmeasured, then set its weight to zero (see “Output Weights” on page 3-24).
- For an input, designate it as an unmeasured disturbance, then define a custom state estimator that ignores the input (see “Disturbance Modeling and Estimation” on page 3-34).

---

**Note** By default, the Model Predictive Control Toolbox assumes that unspecified plant inputs are manipulated variables, and unspecified outputs are measured. Thus, if you didn't specify signal types in the above example, the controller would have four inputs (assuming all plant outputs were measured) and five outputs (assuming all plant inputs were manipulated variables).

---

**Example.** For model CSTR, the default Model Predictive Control Toolbox assumptions are incorrect. You must set its `InputGroup` and `OutputGroup` properties, as illustrated in the above code, or modify the default settings when you load the model into the design tool.

The Model Predictive Control Toolbox provides a “helper function” called `setmpcsignals` to make type definition more convenient. For example

```
CSTR = setmpcsignals(CSTR, 'UD', 2, 'UO', 2);
```

sets `InputGroup` and `OutputGroup` to the same values as in the previous example. The CSTR display would then include the following lines:

```
Input groups:
      Name      Channels
Unmeasured      2
Manipulated      1
```

```
Output groups:
      Name      Channels
Unmeasured      2
Measured         1
```

Notice that `setmpcsignals` sets unspecified inputs to `Manipulated` and unspecified outputs to `Measured`.

See the Control System Toolbox documentation for additional information on LTI object properties.

## Multiinput Multioutput (MIMO) Plants

Most Model Predictive Control Toolbox applications involve plants having multiple inputs and outputs. Model CSTR described in “State-Space Format” on page 2-5 is a MIMO plant, and the state space format extends naturally from single-input single-output (SISO) to MIMO plants.

You can also use the `tf` and `zpk` functions to build a MIMO plant model. For example, consider the following model of a distillation column [11], which has been used in many advanced control studies:

$$\begin{bmatrix} y_1 \\ y_2 \end{bmatrix} = \begin{bmatrix} \frac{12.8e^{-s}}{16.7s + 1} & \frac{-18.9e^{-3s}}{21.0s + 1} & \frac{3.8e^{-8.1s}}{14.9s + 1} \\ \frac{6.6e^{-7s}}{10.9s + 1} & \frac{-19.4e^{-3s}}{14.4s + 1} & \frac{4.9e^{-3.4s}}{13.2s + 1} \end{bmatrix} \begin{bmatrix} u_1 \\ u_2 \\ u_3 \end{bmatrix}$$

Outputs  $y_1$  and  $y_2$  represent measured product purities. The control objective is to hold each at specified setpoints. To do so, the controller manipulates inputs  $u_1$  and  $u_2$ , the flow rates of reflux and reboiler steam, respectively. Input  $u_3$  is a measured feed flow rate disturbance.

The model consists of six transfer functions, one for each input/output pair. Each transfer function is the first-order-plus-delay form often used by process control engineers.

The following code shows how to define the distillation column model for use in the Model Predictive Control Toolbox:

```
g11 = tf( 12.8, [16.7 1], 'IOdelay', 1.0);
g12 = tf(-18.9, [21.0 1], 'IOdelay', 3.0);
g13 = tf( 3.8, [14.9 1], 'IOdelay', 8.1);
g21 = tf( 6.6, [10.9 1], 'IOdelay', 7.0);
g22 = tf(-19.4, [14.4 1], 'IOdelay', 3.0);
g23 = tf( 4.9, [13.2 1], 'IOdelay', 3.4);
DC = [g11 g12 g13
      g21 g22 g23];
DC.InputName = {'Reflux Rate', 'Steam Rate', 'Feed Rate'};
DC.OutputName = {'Distillate Purity', 'Bottoms Purity'};
DC = setmpcsignals(DC, 'MD', 3)
```

The code defines the individual transfer functions, and then forms a matrix in which each row contains the transfer functions for a particular output, and

each column corresponds to a particular input. The code also sets the signal names and designates the third input as a measured disturbance. The resulting LTI object display is as follows:

Transfer function from input "Reflux Rate" to output...

$$\text{Distillate Purity: } \exp(-1*s) * \frac{12.8}{16.7 s + 1}$$

$$\text{Bottoms Purity: } \exp(-7*s) * \frac{6.6}{10.9 s + 1}$$

Transfer function from input "Steam Rate" to output...

$$\text{Distillate Purity: } \exp(-3*s) * \frac{-18.9}{21 s + 1}$$

$$\text{Bottoms Purity: } \exp(-3*s) * \frac{-19.4}{14.4 s + 1}$$

Transfer function from input "Feed Rate" to output...

$$\text{Distillate Purity: } \exp(-8.1*s) * \frac{3.8}{14.9 s + 1}$$

$$\text{Bottoms Purity: } \exp(-3.4*s) * \frac{4.9}{13.2 s + 1}$$

Input groups:

Name	Channels
Measured	3
Manipulated	1,2

Output groups:

Name	Channels
Measured	1,2



## LTI Model Characteristics

The Control System Toolbox provides functions for analyzing LTI models. Some of the more commonly used are listed below. Type the example code at the MATLAB prompt to see how they work for the CSTR example.

<b>Example</b>	<b>Intended Result</b>
<code>dcgain(CSTR)</code>	Calculate gain matrix for the CSTR model's input/output pairs.
<code>impulse(CSTR)</code>	Graph CSTR model's unit-impulse response.
<code>ltiview(CSTR)</code>	Open the LTI Viewer with the CSTR model loaded. You can then display model characteristics by making menu selections.
<code>pole(CSTR)</code>	Calculate CSTR model's poles (to check stability, etc.).
<code>step(CSTR)</code>	Graph CSTR model's unit-step response.
<code>zero(CSTR)</code>	Compute CSTR model's transmission zeros.

## System Identification Toolbox Models

The System Identification Toolbox for MATLAB generates LTI models based on plant input/output data. This section explains how to use such models in the Model Predictive Control Toolbox:

- “System Identification Model Definition Example” on page 2-14
- “Converting a System Identification Toolbox Model to an LTI Object” on page 2-15
- “Step-Response Models” on page 2-17

---

**Note** The System Identification Toolbox is an optional product. To determine whether your installation includes it, type `ver` at the MATLAB prompt, and look for “System Identification Toolbox” in the list of installed products.

---

### System Identification Model Definition Example

To use the System Identification Toolbox, you first create an *iddata* object containing measured values of your plant input and output signals. The following example uses the System Identification Toolbox tutorial data set, a temperature-control application. Load the data as follows:

```
load dryer2
```

This creates vectors `u2` and `y2` in your workspace. Vector `u2` is a sequence of 1000 plant input values (electrical power), and `y2` is the corresponding output sequence (1000 temperature values). The sampling period is 0.08 second.

Create an *iddata* object called `dry` as follows:

```
dry = iddata(y2,u2,0.08);
```

Once the data has been loaded, use the System Identification Toolbox GUI or commands to determine a model that best fits the data. For example, the commands

```
dry.InputName = 'Power';  
dry.OutputName = 'Temperature';  
ze = detrend(dry(1:300));  
m1 = pem(ze);
```

create a System Identification Toolbox model called `m1` (see the System Identification Toolbox documentation for a detailed explanation). If you type

```
whos m1
```

at the MATLAB prompt, the displayed result is

Name	Size	Bytes	Class
m1	4-D	22470	idss object

Notice that `m1` is an `idss` object, one of seven possible System Identification Toolbox model types (`idgrey`, `idarx`, `idpoly`, `idproc`, `idss`, `idmodel`, `idfrd`). The `pem` function settings govern the type of model generated.

Like other System Identification Toolbox objects, `m1` defines a model structure and adjustable parameter values that best fit the data. It also contains System Identification-Toolbox-specific information, such as the algorithm used to estimate the parameters.

## Converting a System Identification Toolbox Model to an LTI Object

Once you've created a System Identification Toolbox model based on your data, you must convert it to a standard LTI object prior to using it in the Model Predictive Control Toolbox. The System Identification Toolbox provides a special conversion function (`ss`).

---

**Note** An exception is the `mpc` function described in “Controller Definition” on page 4-2, which can use System Identification Toolbox models directly.

---

### Creating an LTI State-Space Model

Use the `ss` function to convert a System Identification Toolbox model object (except the `idfrd` type) to a standard Control System Toolbox `ss` (state-space) object, which is the form used internally by the Model Predictive Control Toolbox. For example,

```
m1ss = ss(m1)
```

converts `m1`, an System Identification Toolbox object, to `m1ss`, an LTI `ss` object, and displays the following:

```

a =
      x1      x2      x3
x1    0.9492  -0.2127  0.03679
x2    0.2599   0.6523   0.2342
x3   -0.04822 -0.6639   0.1393

b =
      Power  v@Temperatur
x1   -0.0005008   0.002613
x2   -0.01335   -0.0002421
x3   -0.06729   -0.004463

c =
      x1      x2      x3
Temperature  14.09  -0.108  -0.08164

d =
      Power  v@Temperatur
Temperature    0   0.03968

Input groups:
  Name      Channels
Measured    1
Noise      2

Sampling time: 0.08
Discrete-time model

```

We see that `m1ss` is a 3rd-order, discrete-time, state-space model with a sampling time of 0.08, one output (“Temperature”), and two inputs (“Power” and “v@Temperatur”).

### Noise Inputs

The System Identification Toolbox automatically creates a noise input for each output to model the impact of unmeasured disturbances and measurement

noise. In the above example, there is one output (Temperature). Its associated noise input is `v@Temperatur`.

The System Identification Toolbox designates the noise inputs using the LTI model's `InputGroup` property. In the above example, channel 2 (`v@Temperatur`) is classified as `Noise`, while channel 1 (`Power`) is `Measured`.

When you use such a model in the Model Predictive Control Toolbox, `Noise` inputs will be treated as unmeasured disturbances, and `Measured` inputs will be treated as manipulated variables. (See “Overview” on page 2-2 for discussion of input types.) If these defaults are inappropriate, you must correct them prior to using the model in a design. If you are using the design tool, “Loading a Plant Model” on page 3-3 shows how to specify signal types. When using commands, you need to set the model's `InputGroup` and `OutputGroup` properties, as illustrated in “LTI Object Properties” on page 2-7.

---

**Note** As discussed in “Disturbance Modeling and Estimation” on page 3-34, unmeasured disturbance inputs influence the default controller design. A System Identification Toolbox model of its `Noise` inputs might fit the given data, but another experiment might yield a very different noise model. If a controller designed using such a model seems to work well during setpoint changes but is slow to eliminate disturbances or exhibits steady-state error, try modifying the Model Predictive Control Toolbox disturbance modeling settings.

---

## Step-Response Models

Early predictive control implementations used finite step-response or finite impulse-response models, often called *nonparametric* LTI models (see [6] for example). Such models are easy to determine from plant data ([3], [7]) and they have intuitive appeal.

The System Identification Toolbox includes tools for nonparametric model identification. See its documentation for details.

For example, given the `iddata` object `ze` (defined in “System Identification Model Definition Example” on page 2-14), you could type

```
m2 = step(ze);
```

This identifies a finite step-response model, `m2`, which is a System Identification Toolbox `idarx` object. Then typing

```
step(m2)
```

would display its unit-step response.

You could convert `m2` to an LTI object for use in Model Predictive Control Toolbox design (see “Creating an LTI State-Space Model” on page 2-15). The disadvantage is that the model will be high-order, especially if your plant is MIMO. For example, converting `m2` generates a `ss` object of order 69.

High-order models can degrade certain Model Predictive Control Toolbox operations, such as estimator design. As is the case with most recent predictive control implementations (see [5] for example), the Model Predictive Control Toolbox algorithms work best with a low-order parametric model. For example, reference [10] describes a systematic approach that identifies a step-response model as an intermediate step. The System Identification Toolbox documentation also advocates this approach.

## Using Simulink to Develop LTI Models

The LTI and System Identification Toolbox models discussed in the previous sections are *linear* dynamic models. Most real systems are nonlinear. If you'd like to simulate Model Predictive Control Toolbox control of a nonlinear system, you must model the plant in Simulink.

Although a Model Predictive Control Toolbox controller can regulate a nonlinear plant, the model used within the controller must be linear. In other words, the controller employs a linear approximation of the nonlinear plant. The accuracy of this approximation is a key issue affecting controller performance.

This raises the question: how to obtain the approximation? The usual approach is to *linearize* the nonlinear plant at a specified *operating point*. The Simulink environment provides two ways to accomplish this:

- “Linearization Using Simulink Control Design” on page 2-19
- “Linearization Using Simulink Functions” on page 2-24

### Linearization Using Simulink Control Design

Simulink Control Design is an optional product that supports model linearization. To determine whether or not your license includes it, type `ver` at the MATLAB prompt, and note whether or not “Simulink Control Design” appears in the resulting product list.

The Simulink Control Design documentation includes extensive background on linearization and several examples. You can linearize a Simulink model of the plant alone, or a model that includes both the plant and its controller.

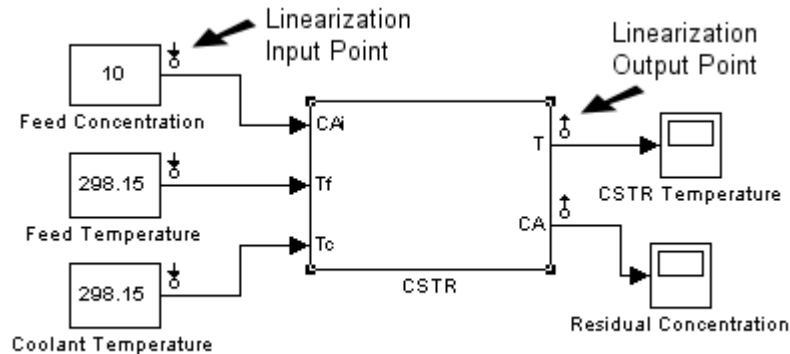
Figure 2-3 is a Simulink model of a continuous stirred-tank reactor (CSTR). It is similar to the one in Figure 2-2, except that here the model is nonlinear and includes an additional input: the feed temperature. This code is stored in the Model Predictive Control Toolbox demo folder. You can open it by typing

```
CSTR_0L
```

at the MATLAB prompt.

As shown in Figure 2-3, the three inputs are being held at constant values: 10 kmol/m<sup>3</sup> for the feed concentration, and 298.15 K for the feed and coolant temperatures. Like the model of Figure 2-2, there are two state variables: the

reactor temperature and the reactant concentration leaving the reactor. The Simulink model defines their initial conditions to be 311.27 K and 8.57 kmol/m<sup>3</sup>, which are at steady state (or equilibrium condition) for the given inputs. (If you run the simulation, the outputs will stay at their initial conditions.)



**Figure 2-3: Simulink Model of Nonlinear Chemical Reactor**

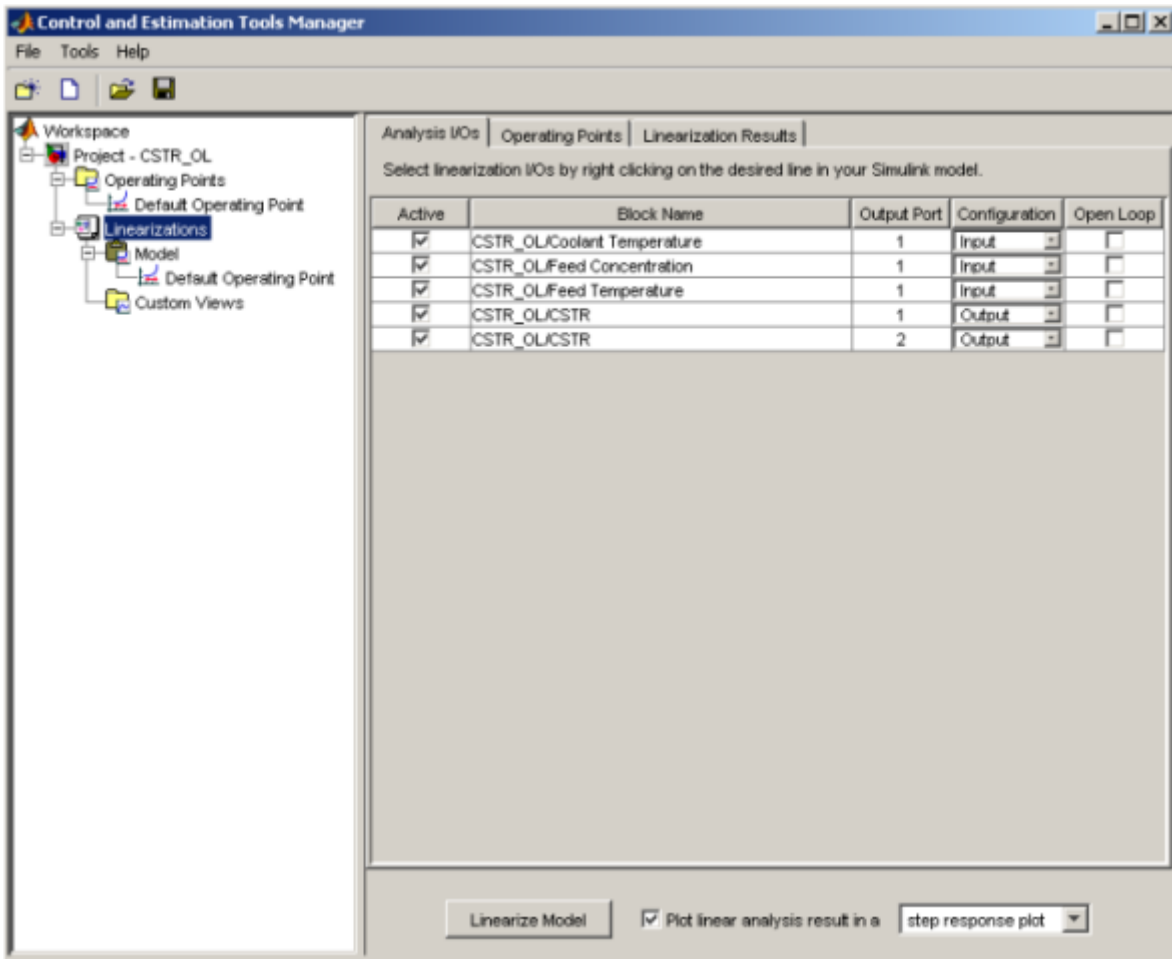
**Note** “Nonlinear Plants” on page 3-57 shows how to linearize this model when a Model Predictive Control block is included.

To linearize Figure 2-3, first designate the input and output signals to be retained in the linear approximation. In general, you would choose signals that will be connected to a controller. In Figure 2-3, all the signals have been selected by adding *linearization points*, i.e., by right-clicking on a signal and selecting either **Input Point** or **Output Point** from the **Linearization Points** submenu.

Next, create a linearization project within the Simulink Control and Estimation Tools Manager. From the **Tools** menu of the Simulink model, select **Control Design/Linear Analysis**.

Figure 2-4 shows the resulting window for the CSTR example. The tool automatically defines the **Default Operating Point** entry, in this case the initial steady-state condition.





**Figure 2-4: CSTR Model Linearization Using Simulink Control Design**

### Linearization at a Specified Operating Point

To calculate the linear approximation at a particular operating point, select **Linearizations** in the tree (highlighting it as shown in Figure 2-4), select the desired operating point on the **Operating Points** tab (if more than one have been defined), and then click the **Linearize Model** button.

Figure 2-4 shows the tool's state after a linearized model has been created at the default operating point. This model appears in the tree as the icon labeled **Model**. The tree has been expanded to show its associated operating point, which in this case is labeled **Default Operating Point**. You can select the model in the tree, right-click, and then export it to MATLAB, making it available for use in another tool.

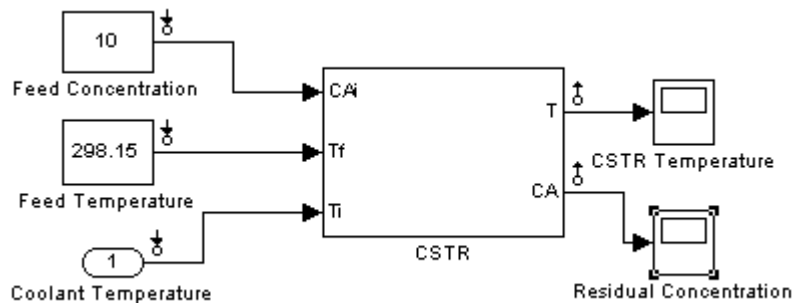
### Determining a New Operating Point

You are likely to need to modify a nonlinear model's operating point. For example, the above CSTR model has a poor reactant conversion. The feed contains  $10 \text{ kmol/m}^3$ , and the residual is  $8.57 \text{ kmol/m}^3$ , so only 1.43 has reacted (14.3% conversion).

Suppose that you'd like to react 80% (i.e., a residual concentration of  $2 \text{ kmol/m}^3$ ). To increase the conversion you need to increase the CSTR temperature, but how much? Also, to change the CSTR temperature you need to change the coolant temperature, but how much?

One approach would be to change the coolant temperature, run a simulation of sufficient duration to reach a new steady state, check the final residual concentration, and repeat until you achieve the desired  $2.0 \text{ kmol/m}^3$  residual. This is tedious and essentially impossible in a more complex situation where you are trying to match several targets simultaneously.

Simulink Control Design can search for a new steady state operating point that achieves the desired conversion. First, you must modify the model so Simulink can change the coolant temperature. One way is to represent the coolant temperature with an Inport block, as shown below (compare to Figure 2-3, which uses a Constant block).



Save this modified model under a new name. Then from the **Tools** menu, select **Control Design/Linear Analysis** as before. If the Control and Estimation Tools Manager window containing CSTR\_OL is still open (as shown in Figure 2-4), a new linearization project will be inserted. Otherwise, the window will open with CSTR\_OL as a new project.

As before, it will contain a default operating point, which is as before except that the coolant temperature appears as an input and defaults to zero. To modify this, select **Operating Points** in the tree, and select the **Create Operating Points** tab. On this panel, select the **States** tab and set the check boxes as shown below.

State	Value	State Specifications			
		<input type="checkbox"/> Known	<input type="checkbox"/> Steady State	Minimum	Maximum
<b>CSTR_IN/CSTR/Integrator</b>					
State - 1	311.267	<input type="checkbox"/>	<input checked="" type="checkbox"/>	0	Inf
<b>CSTR_IN/CSTR/Integrator1</b>					
State - 1	2	<input checked="" type="checkbox"/>	<input checked="" type="checkbox"/>	0	Inf

Also set the desired value of the second state (the residual concentration) to 2, as shown. You are asking for a new operating point in which one state is specified (known) and both are at steady state. The reactor temperature **value** (311.267) is an initial guess, and the tool uses optimization to find the value that satisfies all the specifications.

Next, click the **Inputs** tab and verify that the coolant temperature input is not specified (i.e., its **Known** check box is not selected) as shown below.

Input	Value	Input Specifications		
		<input type="checkbox"/> Known	Minimum	Maximum
<b>CSTR_IN/Coolant Temperature</b>				
Channel - 1	298	<input type="checkbox"/>	-Inf	Inf

The **value** is an initial guess that will be changed. You can set it to 298 to help the tool converge its trial-and-error calculations. (The default guess of 0 should

also work in this case, but in general you might have to supply a better guess to aid convergence.)

Finally, click the **Compute Operating Point** button at the panel bottom. A calculation progress panel shows the specification error at each iteration. When it's finished, you should see the line, "Operating point specifications were successfully met" and a new operating point should appear in the tree. Click on this and observe that the required reactor temperature is 373.13 K, and the required coolant temperature is 305.20 K.

Let's calculate a new linearized model at this condition, comparing it to that obtained at the original point. Suppose you exported the original model as `Model11`, and the other as `Model12`. The following command would compare their step responses:

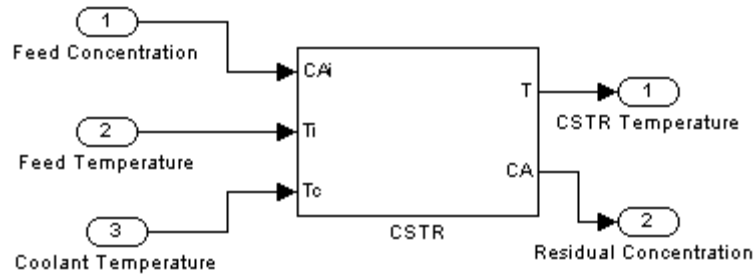
```
step(Model11, Model12)
```

You should see some significant quantitative and qualitative differences, especially in the response to a change in feed concentration. At the original low-conversion state, increasing the feed concentration increases both the reactor temperature and the residual concentration. At high conversion, however, the reaction is more sensitive to temperature changes. Increasing the feed concentration causes an initial rise in the residual concentration, but the increased temperature accelerates the reaction rate and the residual concentration goes *below* its initial value. Thus, if one were trying to control conversion by adjusting the feed concentration, a model-based controller designed for low conversion would be certain to fail at high conversion.

### Linearization Using Simulink Functions

Another approach is to linearize the model using standard Simulink functions. This is more restrictive: you cannot perform open loop analysis of the Simulink model, and the signals to be retained in the linearized model must be connected to an inport or outport block. On the other hand, Simulink Control Design is not needed.

The diagram below shows a modification of Figure 2-3 with three inport blocks designating the input signals (on the left-hand side of the model) and two outports designating the outputs (on the right).



Suppose this model were named CSTR\_INOUT. The `linmod` command linearizes it as follows:

```
[a,b,c,d]=linmod('CSTR_INOUT')
```

a =

```
-0.2505    1.9897
-0.0880   -1.1669
```

b =

```
    0    1.0000    0.3000
  1.0000    0    0
```

c =

```
  1.0000    0
    0    1.0000
```

d =

```
    0    0    0
    0    0    0
```

By default, `linmod` uses the initial conditions defined in the model as the operating point. Options allow you to specify an operating point. The command outputs are the standard state-space matrices defining an LTI model. You can use these to create an LTI model as follows:

```
cstr = ss(a,b,c,d)
```

## Bibliography

- [1] Allgower, F., and A. Zheng, *Nonlinear Model Predictive Control*, Springer-Verlag, 2000.
- [2] Camacho, E. F., and C. Bordons, *Model Predictive Control*, Springer-Verlag, 1999.
- [3] Cutler, C., and F. Yocum, "Experience with the DMC inverse for identification," *Chemical Process Control — CPC IV* (Y. Arkun and W. H. Ray, eds.), CACHE, 1991.
- [4] Kouvaritakis, B., and M. Cannon, *Non-Linear Predictive Control: Theory & Practice*, IEE Publishing, 2001.
- [5] Maciejowski, J. M., *Predictive Control with Constraints*, Pearson Education POD, 2002.
- [6] Prett, D., and C. Garcia, *Fundamental Process Control*, Butterworths, 1988.
- [7] Ricker, N. L., "The use of bias least-squares estimators for parameters in discrete-time pulse response models," *Ind. Eng. Chem. Res.*, Vol. 27, pp. 343, 1988.
- [8] Rossiter, J. A., *Model-Based Predictive Control: A Practical Approach*, CRC Press, 2003.
- [9] Seborg, D. E., T. F. Edgar, and D. A. Mellichamp, *Process Dynamics and Control*, 2nd Edition, Wiley, 2004, pp. 34-36 and 94-95.
- [10] Wang, L., P. Gawthrop, C. Chessari, T. Podsiadly, and A. Giles, "Indirect approach to continuous time system identification of food extruder," *J. Process Control*, Vol. 14, Number 6, pp. 603-615, 2004.
- [11] Wood, R. K., and M. W. Berry, *Chem. Eng. Sci.*, Vol. 28, pp. 1707, 1973.





# The Design Tool

---

Introduction (p. 3-2)	Opens the design tool and navigates its views
Linear Simulations (p. 3-12)	Defines and runs simulation scenarios
Changing Controller Settings (p. 3-20)	Modifies the default controller settings for improved close-loop performance
Defining Soft Output Constraints (p. 3-42)	Includes hard and soft output constraints in your controller specifications
Robustness Testing (p. 3-46)	Tests sensitivity to prediction errors
Plant Models with Delays (p. 3-49)	Defines controller settings for plants having delays
Nonsquare Plants (p. 3-55)	Designs controllers for plants with unequal numbers of manipulated variables and outputs
Nonlinear Plants (p. 3-57)	Works with Simulink models – linearization and nominal conditions
Saving Your Work (p. 3-69)	Exports your controller design and saves your design specifications
Loading Your Saved Work (p. 3-72)	Imports a controller and loads saved design specifications

# Introduction

The Model Predictive Control Toolbox design tool is a graphical user interface for controller design. It operates under the Control and Estimation Tools Manager. (For another use of the Control and Estimation Tools Manager, see “Linearization Using Simulink Control Design” on page 2-19.)

This section covers the following topics:

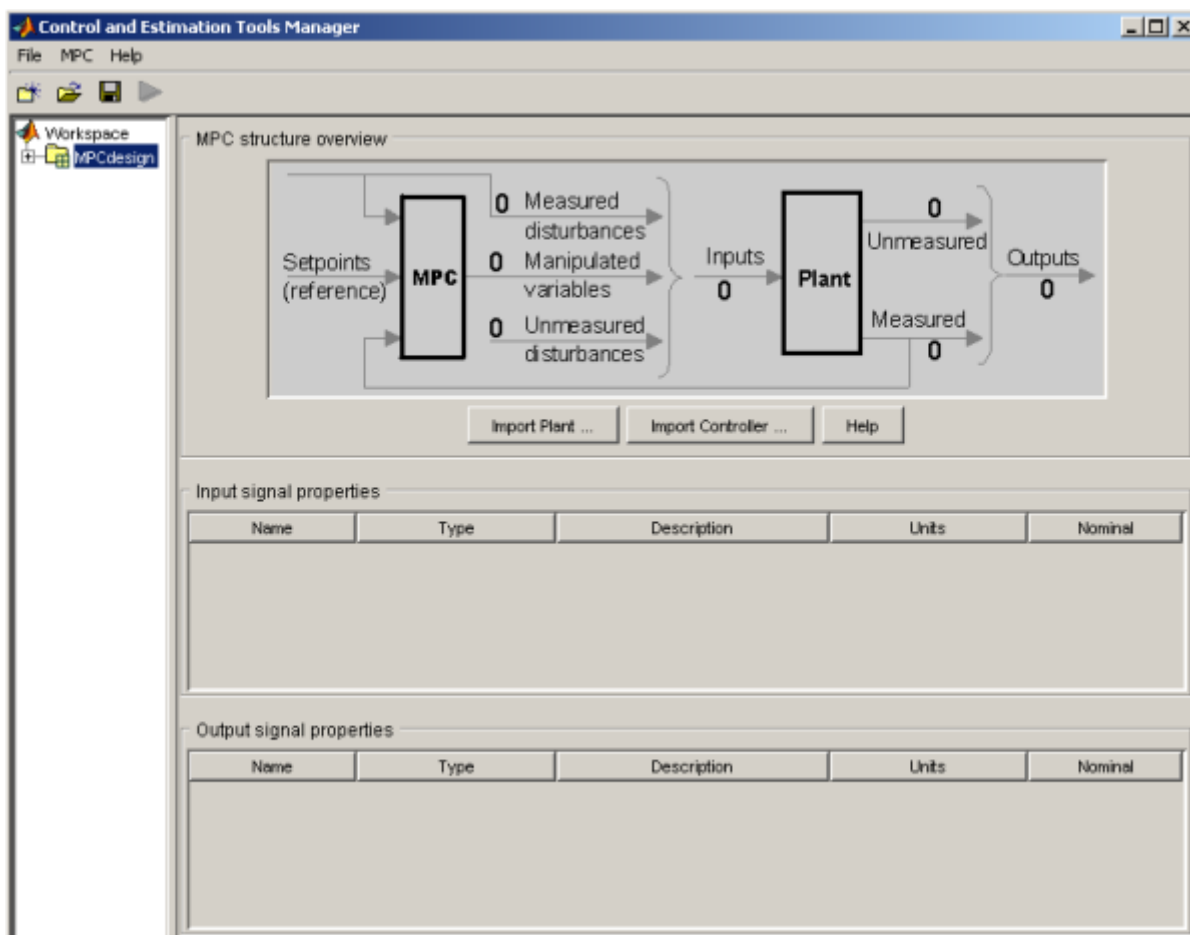
- “Starting the Design Tool” on page 3-2
- “Loading a Plant Model” on page 3-3
- “Signal Property Specifications” on page 3-6

## Starting the Design Tool

Start the design tool by typing the MATLAB command

```
mpctool
```

The **Control and Estimation Tools Manager** window appears, as shown below. By default, it contains a Model Predictive Control Toolbox task called **MPCdesign** (listed in the tree view on the left side of the window), which is selected, causing the view shown on the right to appear.



**Figure 3-1: Model Predictive Control Toolbox Design Tool Initial View**

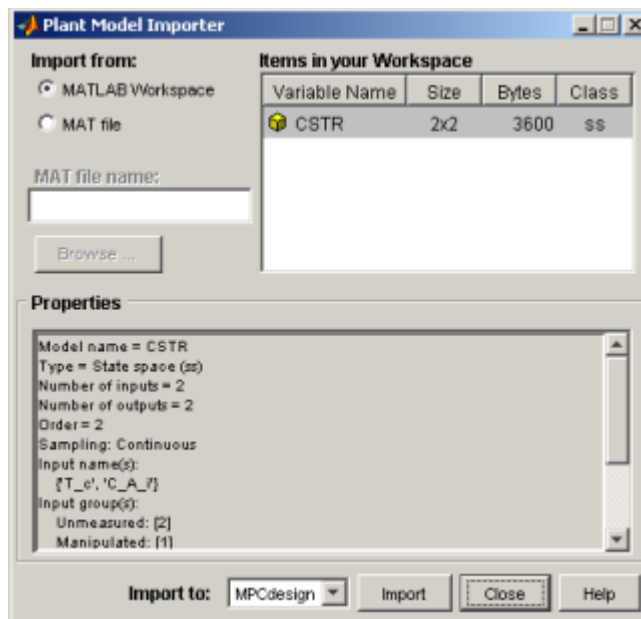
## Loading a Plant Model

The first step in the design is to load a plant model. Its dimensions and signal characteristics set the context for the remaining steps. You can either load the model directly, as described in this section, or indirectly by importing a controller or a saved design (see “Loading Your Saved Work” on page 3-72).

The following example uses the CSTR model described in “State-Space Format” on page 2-5. Verify that the LTI object CSTR is in your MATLAB workspace (if necessary, create the model as explained in “State-Space Format” on page 2-5, and set its label and signal type properties as explained in “LTI Object Properties” on page 2-7).

### Model Import Dialog

Click the **Import Plant** button in the design tool’s initial view (see Figure 3-1). The **Plant Model Importer** dialog appears.

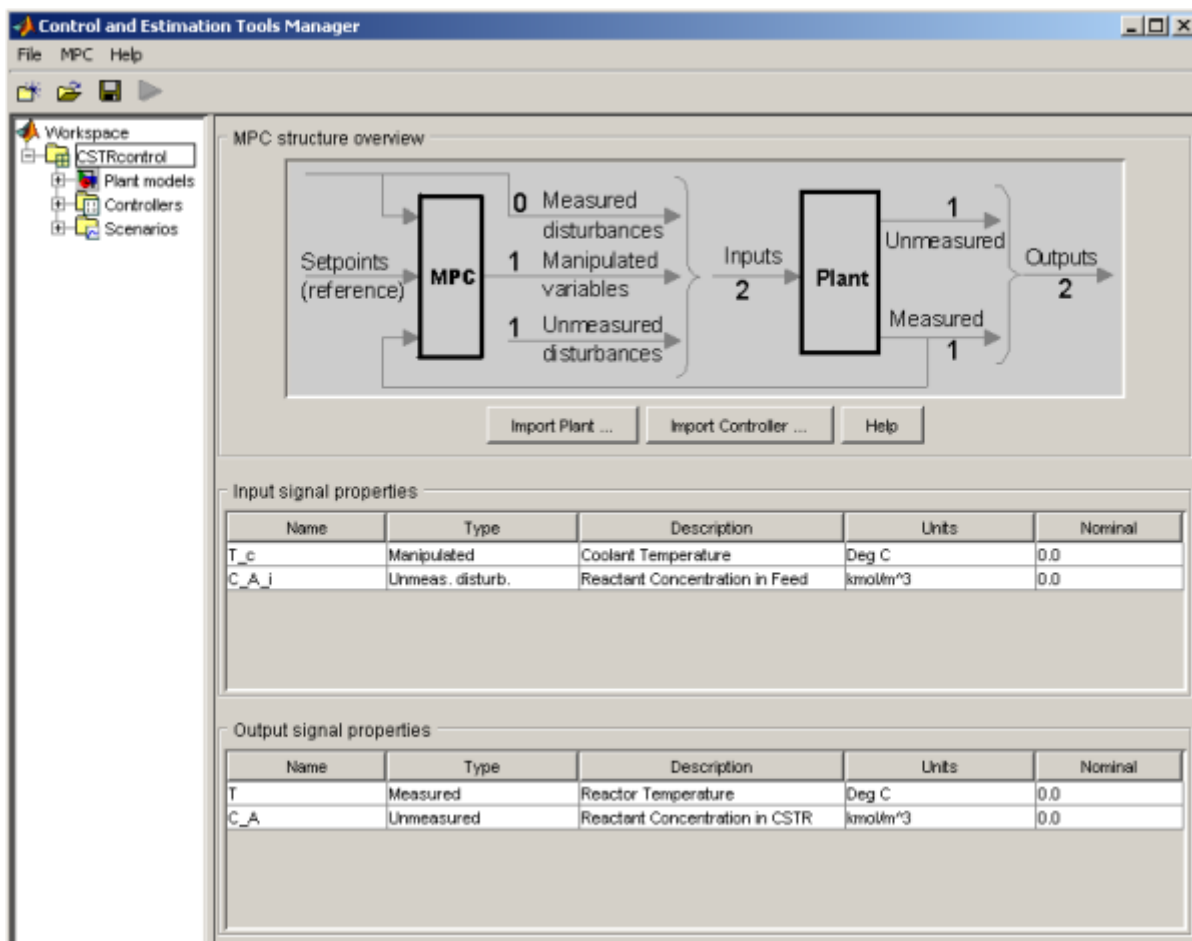


**Figure 3-2: Plant Model Importer Dialog**

The **Import from MATLAB Workspace** radio button should be selected by default, as shown. The dialog section labeled **Items in your Workspace** lists your LTI models. If CSTR doesn’t appear, define it as discussed above, then reactivate the import dialog.

Once CSTR appears, select it. The dialog section labeled **Properties** then displays the number of inputs and outputs, their names and signal types, etc.

Click the **Import** button. This loads CSTR into the design tool. Then click the **Close** button (otherwise the dialog remains visible in case you want to import another model). The design tool should appear as in Figure 3-3.



**Figure 3-3: The Model Predictive Control Toolbox Design Tool's Signal Definition View**

### Signal Property Specifications

The figure's graphical display indicates that you've imported a plant model by showing the number of inputs and outputs, and the number in each subclass: measured disturbance, manipulated variables, etc.). Also, the tables labeled **Input signal properties** and **Output signal properties** fill with data:

- The **Name** entries are from the CSTR model's InputName and OutputName properties (the design tool assigns defaults if necessary). You can edit these at any time.
- The **Type** entries are from the CSTR model's InputGroup and OutputGroup properties. (The design tool defaults all unspecified inputs to manipulated variables and all unspecified outputs to measured.)

---

**Note** Once you leave this view, if you subsequently change a signal type, you will have to restart the design. Be sure the signal types are correct at the beginning.

---

- The **Description** and **Unit** entries are optional. You can enter the values shown in Figure 3-3 manually. As you will see, the design tool uses them to label plots and other tables.
- The **Nominal** entries are initial conditions for simulations. The design tool default is 0.0.

## Navigation Using the Tree View

The *tree* in the left-hand frame of Figure 3-3 shows that the default **MPCdesign** node (see Figure 3-1) has been renamed **CSTRcontrol** by clicking on the name, waiting for the usual edit box to appear, typing the new name, and pressing **Enter** to finalize the choice.

Figure 3-3 also shows three new nodes below **CSTRcontrol**. These activate once you've imported a plant model (or controller).

In general, clicking a node displays a *view* supporting a particular design activity.

### The Project View (Signal Properties Tables)

For example, clicking the *project* node (**CSTRcontrol** in Figure 3-3) allows you to review and edit the signal properties tables.

### Listing Your Plant Models

Select the **Plant models** node to list the plant models you've imported, as shown in Figure 3-4. (Each model name is editable.) The **Model details** section displays properties of the selected model. There is also a space to enter notes describing the model's special features. Buttons allow you to import a new model or delete one you no longer need.

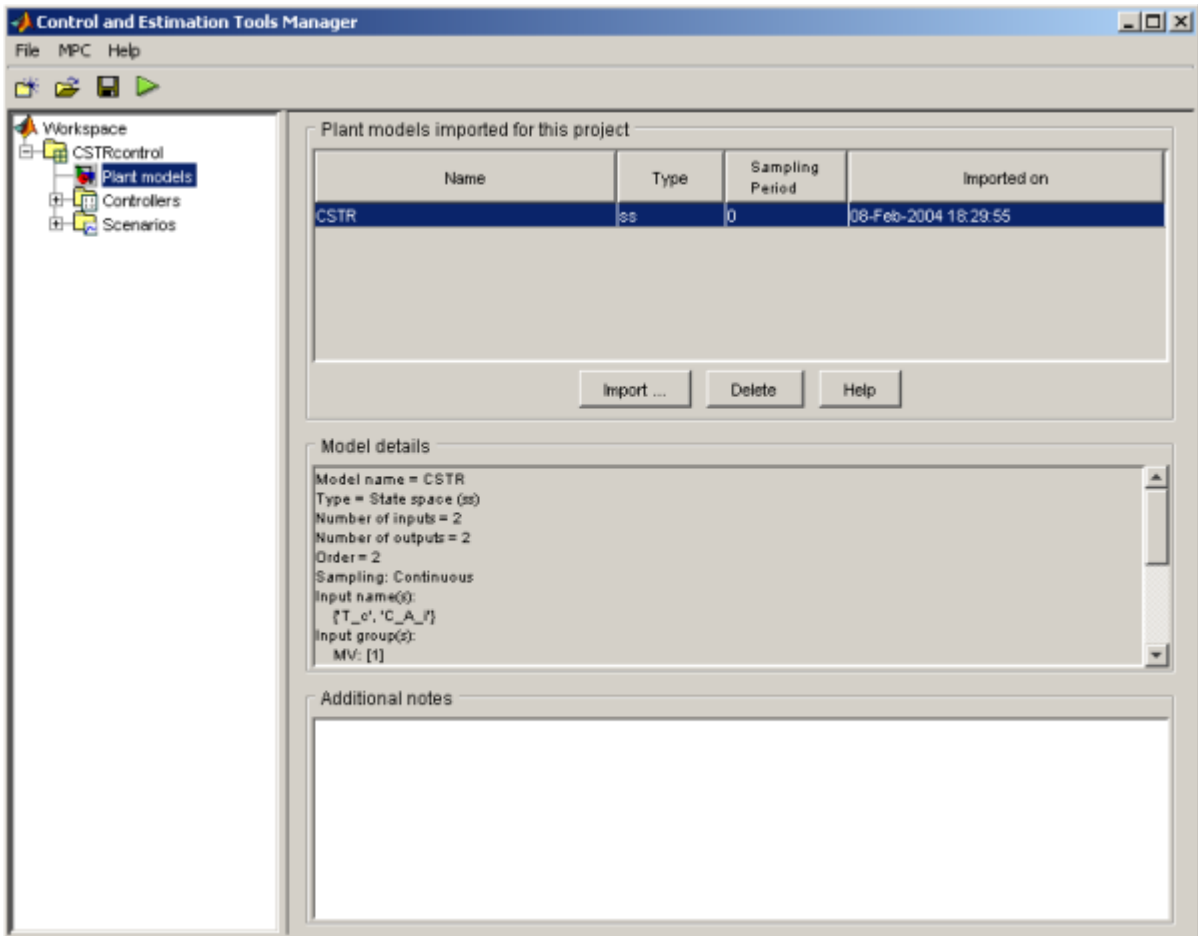


Figure 3-4: Plant Models View with CSTR Model Selected



## Viewing Your Controllers

Next, select **Controllers**. The view shown in Figure 3-5 appears. A + sign to the left of **Controllers** indicates that it contains subnodes. You can click on a + sign to expand the tree (as shown in Figure 3-5, where the + sign has changed to a - sign).

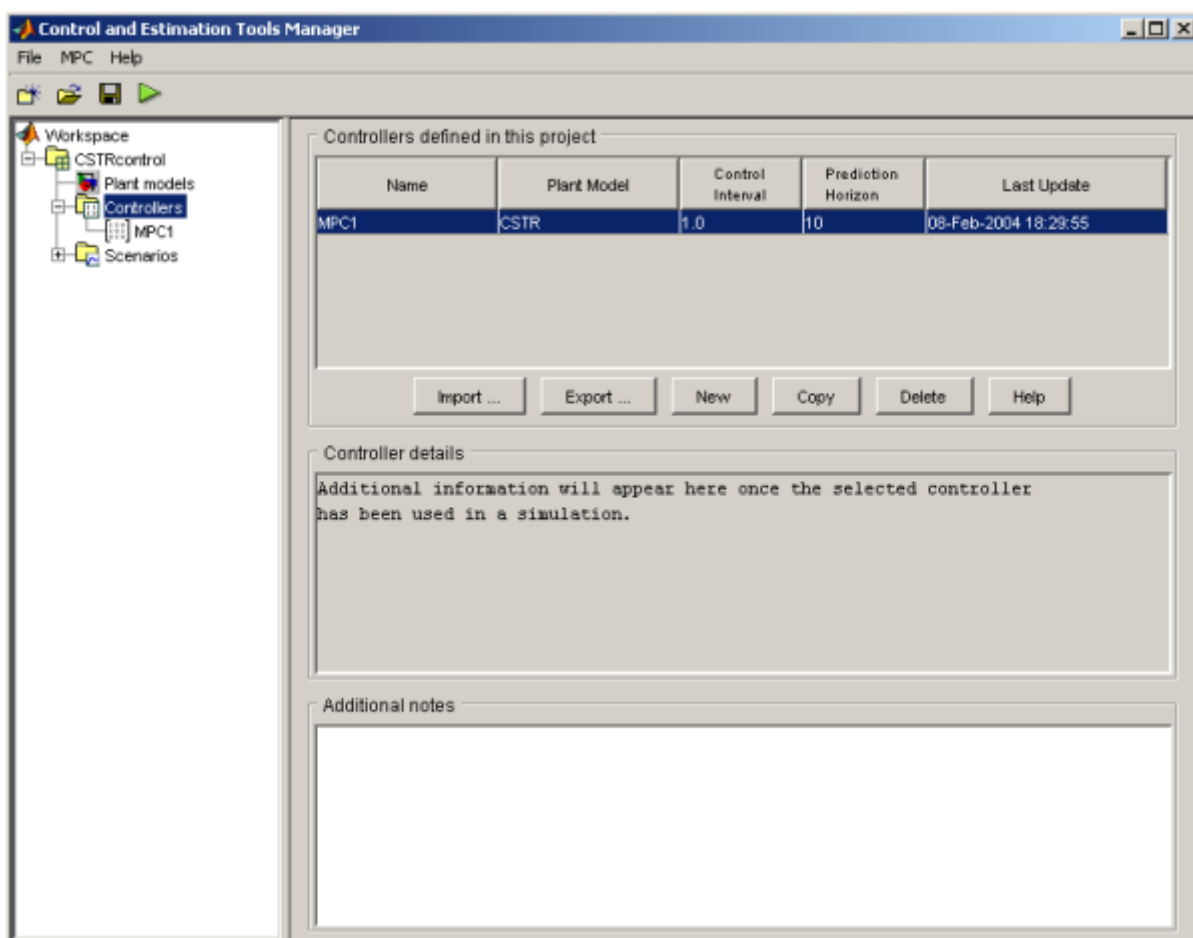


Figure 3-5: Controllers View

The table at the top of Figure 3-5 lists all the controllers you've defined. The design tool automatically creates a controller containing the Model Predictive Control Toolbox defaults, naming it **MPC1**. It is a subnode of **Controllers**.

---

**Note** If you define additional controllers, they will appear here. For example, you might want to test several options, saving each as a separate controller, making it easy to switch from one to another during testing.

---

The table **Controllers defined in this project** allows you to edit the controller name and gives you quick access to three important design parameters: **Plant Model**, **Control Interval**, and **Prediction Horizon**. All are editable, but leave them at their default values for this example.

The buttons shown in Figure 3-5 let you do the following:

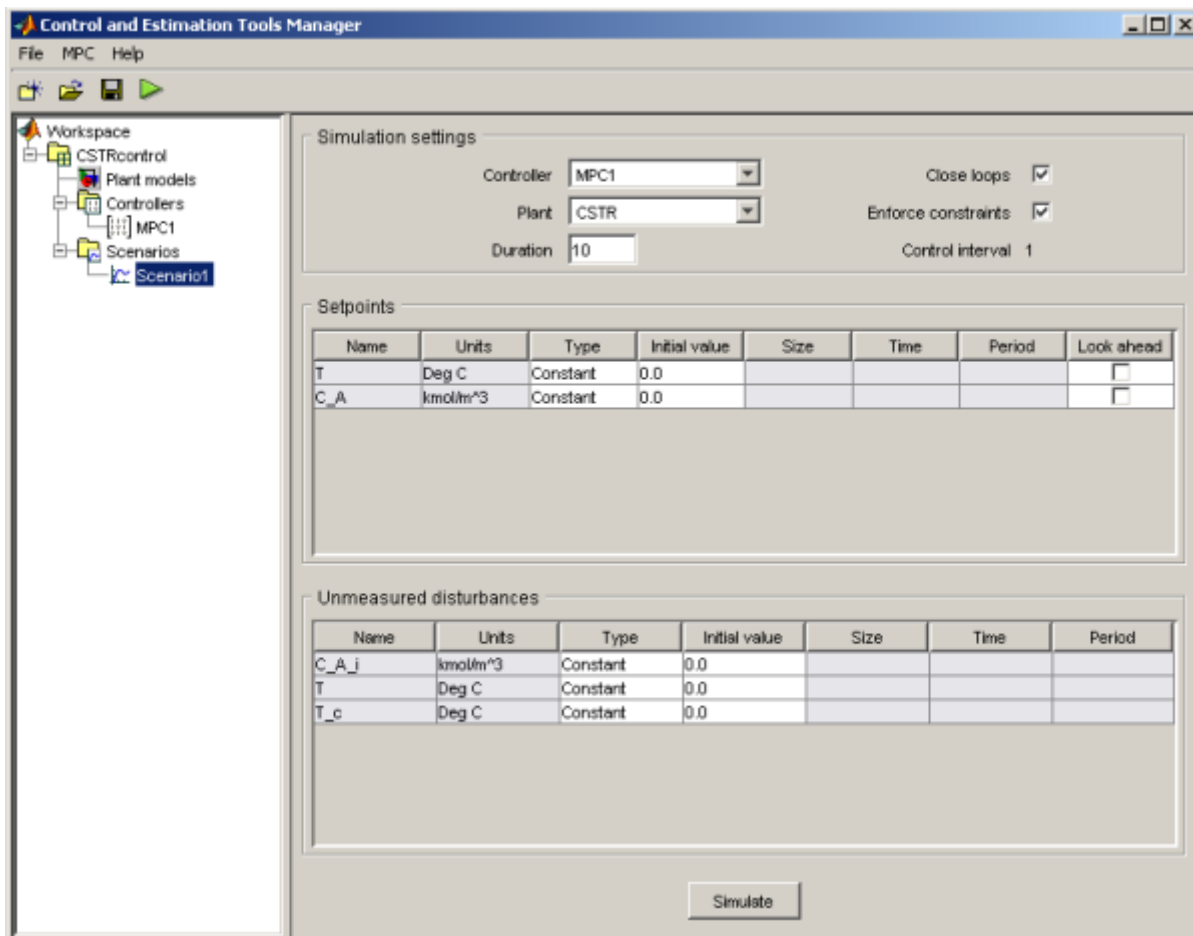
- Import a controller designed previously and stored either in your workspace or in a MAT-file
- Export the selected controller to your workspace
- Create a new controller initialized to the Model Predictive Control Toolbox defaults
- Copy the selected controller, creating a duplicate you can modify
- Delete the selected controller

You can also right-click on the **Controllers** node to access menu options **New**, **Import**, and **Export**, or on one of its subnodes to access menu options **Copy**, **Rename**, **Export**, and **Delete**.

Select the **MPC1** node to display the Model Predictive Control Toolbox default controller settings. (“Changing Controller Settings” on page 3-20 covers this view in detail).

### Viewing Simulation Scenarios

A *scenario* is a set of conditions defining a simulation. The design tool creates a default scenario and names it **Scenario1**. To view it, click on the + symbol next to the **Scenarios** node, and select the **Scenario1** subnode. You should see a view like that shown in Figure 3-6.



**Figure 3-6: Scenario1 Simulation Parameters**

Whenever you select the **Scenarios** node, you see a table summarizing your current scenarios (not shown). Its function is similar to the **Controllers** view described above.

## Linear Simulations

You will usually want to test your controller in simulations. The Model Predictive Control Toolbox design tool makes it easy to run closed-loop simulations involving a Model Predictive Control Toolbox controller and an LTI plant model. This plant can differ from that used in the controller design, allowing you to test your controller's sensitivity to prediction errors (see “Robustness Testing” on page 3-46).

This section covers the following topics:

- “Defining Simulation Conditions” on page 3-12
- “Running a Simulation” on page 3-14
- “Open-Loop Simulations” on page 3-16

### Defining Simulation Conditions

To define simulation conditions, select an existing scenario node or create a new one, and then edit its tabular fields to define your conditions.

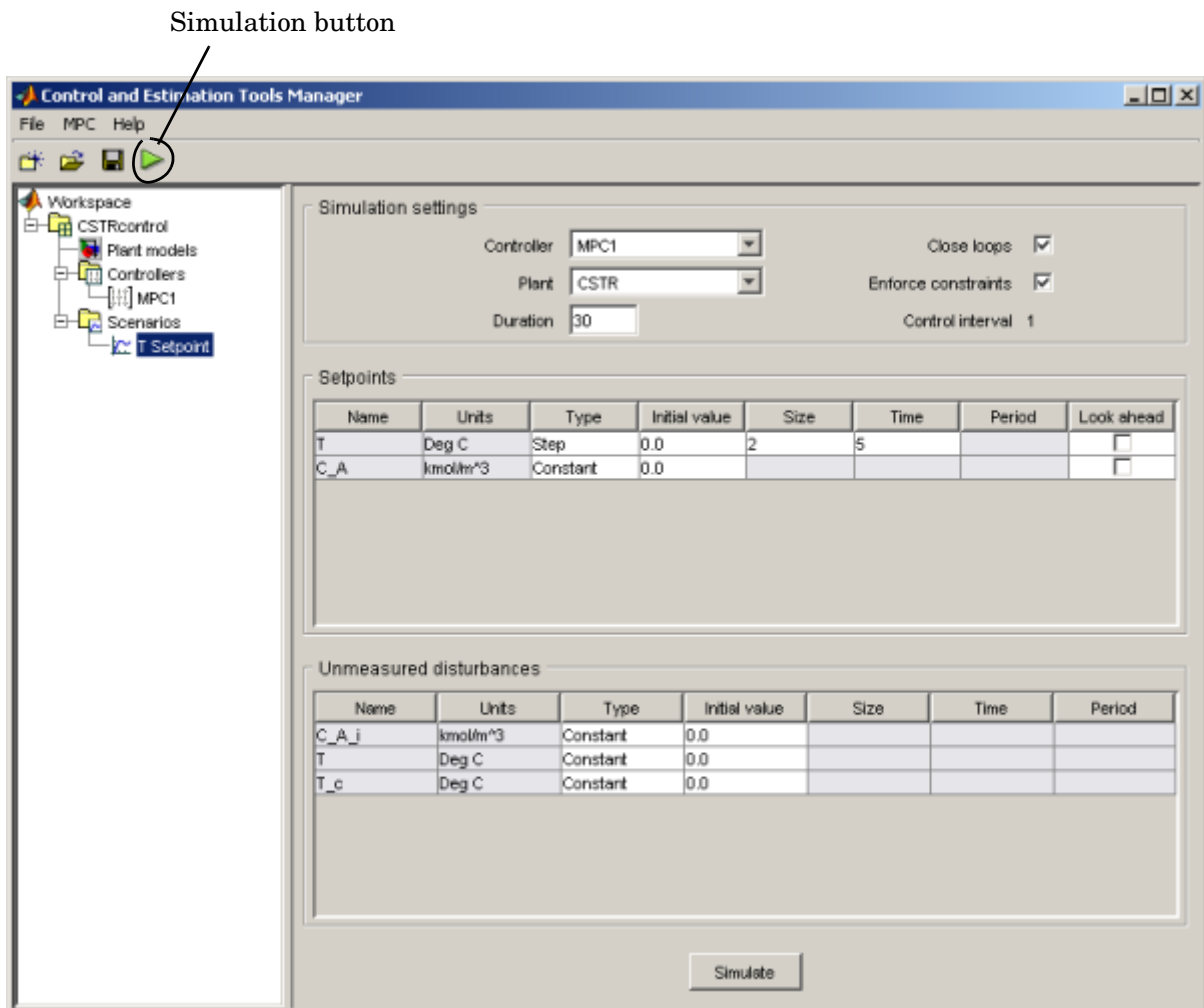
Figure 3-7 shows the result of renaming the default **Scenario1** node to **T Setpoint** and editing its default conditions. The required editing steps are as follows:

- Increase **Duration** from 10 to 30
- Locate the tabular data defining the reactor temperature setpoint (first row of the upper table in Figure 3-7)
  - Click the **Type** table cell and select Step from the list of choices
  - Change **Size** from 1.0 to 2
  - Change **Time** from 1.0 to 5

---

**Note** The **Control Interval** is a property of the controller being used (MPC1 in this case). To change it, select the controller node in the tree, and then edit the value on the **Model and Horizons** tab (see “Model and Horizons” on page 3-20). Such a change would apply to all simulations involving that controller.

---



**Figure 3-7: CSTR Temperature Setpoint Change Scenario**

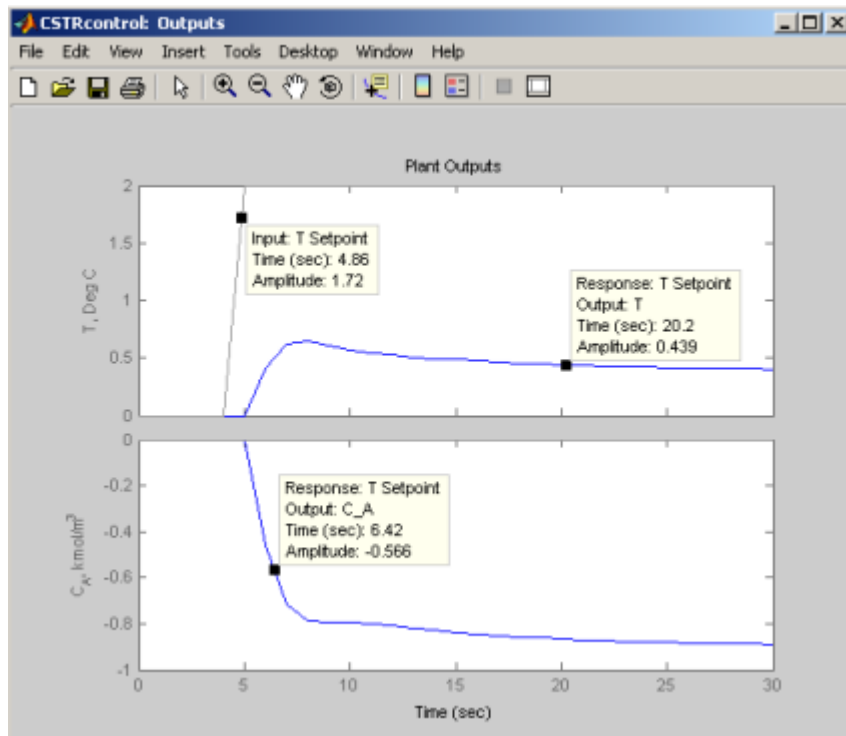
## Running a Simulation

To run a simulation, do *one* of the following:

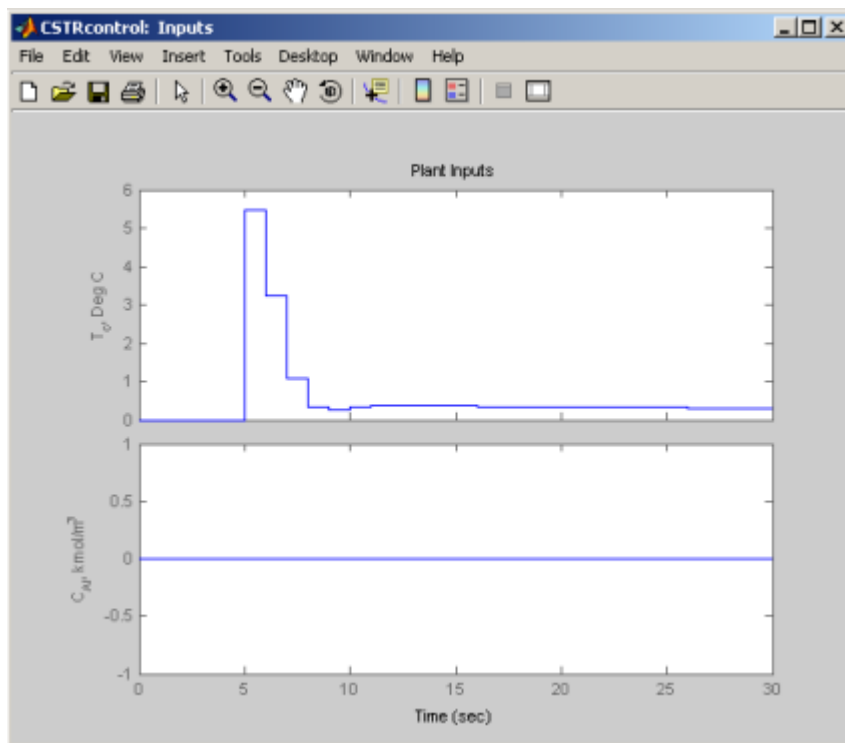
- Select the scenario you want to run, and click its **Simulate** button (see this button near the bottom of Figure 3-7)
- Click the toolbar's Simulation button, the triangular icon shown on the top left of Figure 3-7.

The toolbar method runs the *current scenario*, i.e., the one most recently selected or modified.

Try running the **T Setpoint** scenario. This should generate the two response plot windows shown in Figure 3-8 and Figure 3-9.



**Figure 3-8: Plant Outputs for T Setpoint Scenario**



**Figure 3-9: Plant Inputs for T Setpoint Scenario**

Figure 3-8 shows that the reactor temperature setpoint increases suddenly by 2 degrees at  $t = 5$ , as you specified when defining the scenario in the previous section. Unfortunately, the temperature does not track the setpoint very well, and there is a persistent error of about 1.6 degrees at the end of the simulation.

Also, the controller requests a sudden jump in the coolant temperature (see upper graph in Figure 3-9), which might be difficult to deliver in practice. (The lower graph in Figure 3-9 shows that the feed concentration,  $C_{Ai}$ , remains constant, as specified in the scenario.)

See “Changing Controller Settings” on page 3-20 for ways to overcome these deficiencies.

---

**Note** Figure 3-8 has *data markers*. To add these, left-click on the curve to create the data marker. Drag a marker to relocate it. Left-click in a graph's white space to erase its markers. For more information on data markers, see the Control System Toolbox documentation.

---

## Open-Loop Simulations

By default, scenarios are *closed loop*, i.e., an active controller adjusts the manipulated variables entering your plant based on feedback from the plant outputs. You can also run *open-loop* simulations that test the plant model alone.

For example, you might want to check your plant model's response to a particular input without opening another tool. You might also want to display unmeasured disturbance signals before using them in a closed-loop simulation.

To see how this works, create a new scenario by right-clicking on the **T Setpoint** node in the tree, and selecting **Copy Scenario** in the resulting menu. Rename the copy **OpenLoop**.

Select **OpenLoop** in the tree. On its scenario view, change **Duration** to 100, and turn off (clear) **Close loops**.

Open-loop simulations ignore the **Setpoints** table settings, so there's no need to modify them.

If CSTR had a measured disturbance input, the panel would contain another table allowing you to specify it.

For this example, focus on the **Unmeasured disturbances** table. Configure it as shown below.

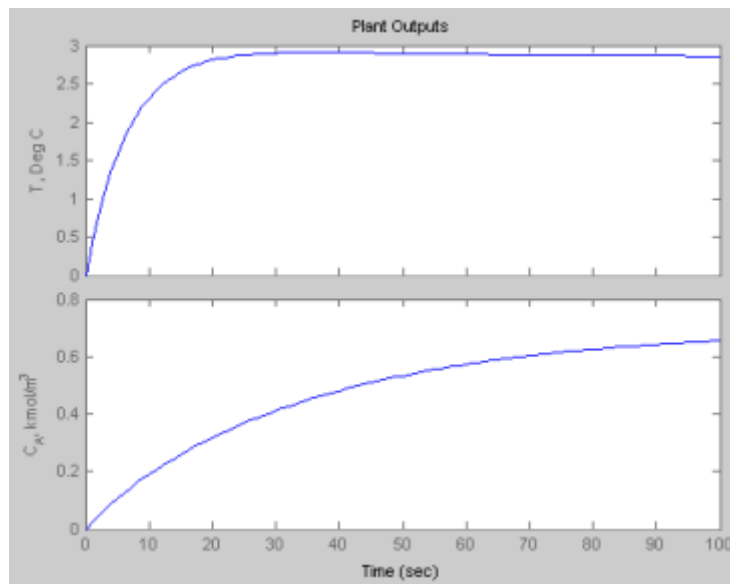
Unmeasured disturbances						
Name	Units	Type	Initial value	Size	Time	Period
C_A_I	kmol/m <sup>3</sup>	Constant	1			
T	Deg C	Constant	0			
T_c	Deg C	Constant	0			



The  $C_{A_i}$  input's nominal value is 0.0 (see Figure 3-3), so the above models a sudden increase to 1 at the beginning of the simulation. The following is an equivalent setup using the Step type.

Unmeasured disturbances						
Name	Units	Type	Initial value	Size	Time	Period
$C_{A_i}$	kmol/m <sup>3</sup>	Step	0	1	0	
T	Deg C	Constant	0			
T <sub>c</sub>	Deg C	Constant	0			

Using one of these, simulate the scenario (click its **Simulate** button). The output response plot should be as shown below.



This is the CSTR model's open-loop response to a unit step in the  $C_{A_i}$  disturbance input.

You could also set up the table as shown below.

Unmeasured disturbances						
Name	Units	Type	Initial value	Size	Time	Period
C_A_J	kmol/m <sup>3</sup>	Constant	0			
T	Deg C	Constant	0			
T_c	Deg C	Constant	1			

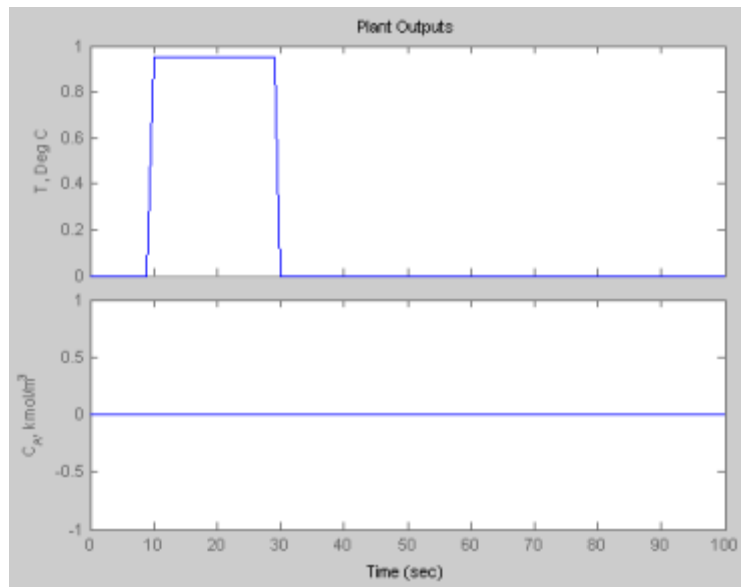
This simulation would display the open-loop response to a unit step in the  $T_c$  manipulated variable input (try it).

Finally, set it up as follows.

Unmeasured disturbances						
Name	Units	Type	Initial value	Size	Time	Period
C_A_J	kmol/m <sup>3</sup>	Constant	0			
T	Deg C	Pulse	0	0.95	10	20
T_c	Deg C	Constant	0			

This adds a pulse to the T output. The pulse begins at time  $t = 10$ , and lasts 20 time units. Its height is 0.95 degrees.

Run the simulation. The output response plot displays the pulse (see Figure 3-10). In this case, the T output's nominal value is zero, so you only see the pulse. (If the T output had a nonzero nominal value, the pulse would add to that.)



**Figure 3-10: Response Plot Showing Open Loop Pulse Disturbance**

If you were to run a *closed-loop* simulation with this same  $T$  disturbance, the controller would attempt to hold  $T$  at its setpoint, and the result would differ from that shown in Figure 3-10.

## Changing Controller Settings

The simulations shown in Figure 3-8 and Figure 3-9 used the default controller settings. These often work well, but the CSTR application is an exception. The following discussion covers the main controller options in more detail, and shows how to tune a controller for better performance.

This section covers the following topics:

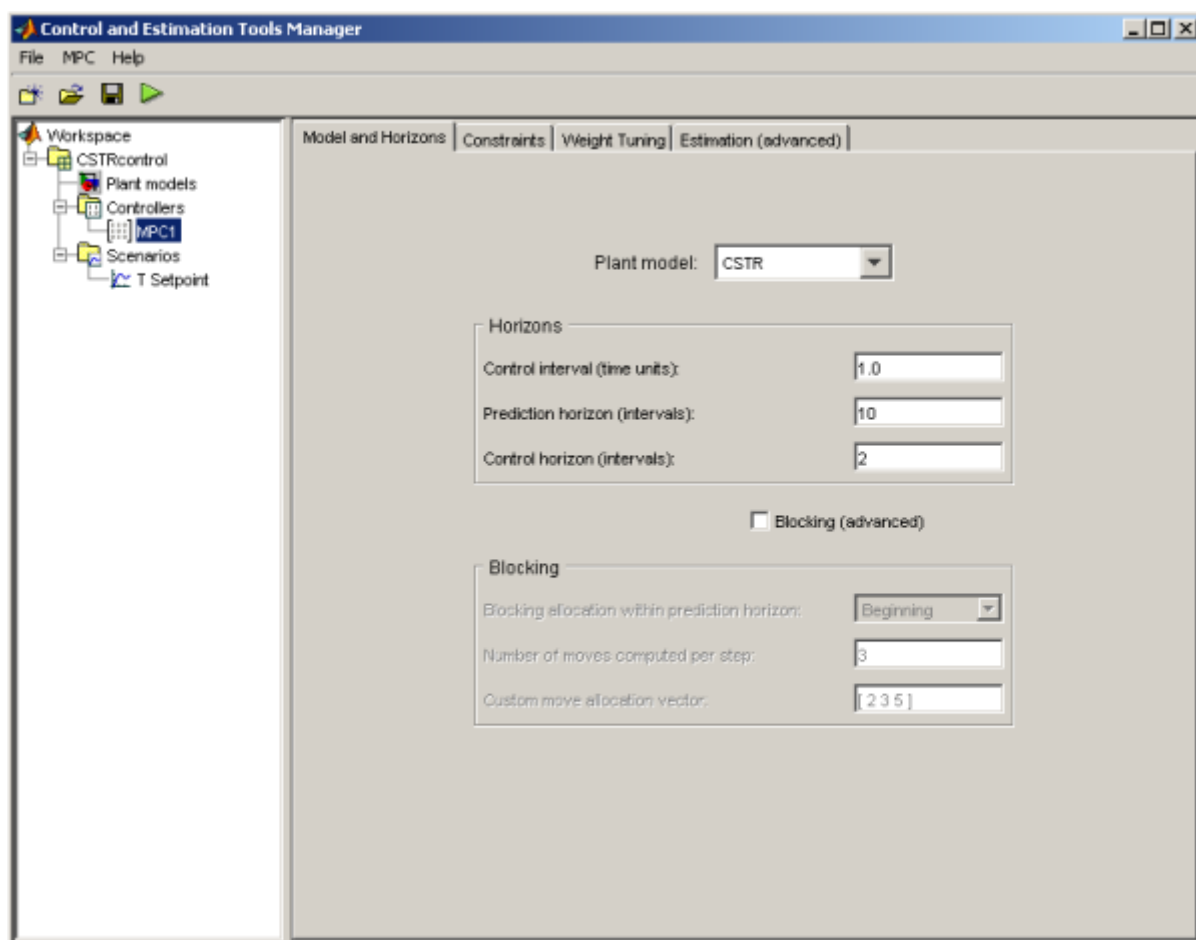
- “Model and Horizons” on page 3-20
- “Weight Tuning” on page 3-22
- “Blocking” on page 3-27
- “Defining Manipulated Variable Constraints” on page 3-30
- “Disturbance Modeling and Estimation” on page 3-34

### Model and Horizons

Select your **MPC1** controller in the tree. The view shown in Figure 3-11 should appear. If necessary, select the **Model and Horizons** tab to bring it to the front. This tab contains the following controller options:

- **Plant model** specifies the LTI model to be used for controller predictions.
- **Control interval** sets the elapsed time between successive adjustments of the controller’s manipulated variables.
- **Prediction horizon** is the number of control intervals over which the outputs are to be optimized.
- **Control horizon** sets the number of control intervals over which the manipulated variables are to be optimized.
- Selecting the **Blocking** option gives you more control over the way in which the controller’s *moves* are allocated.

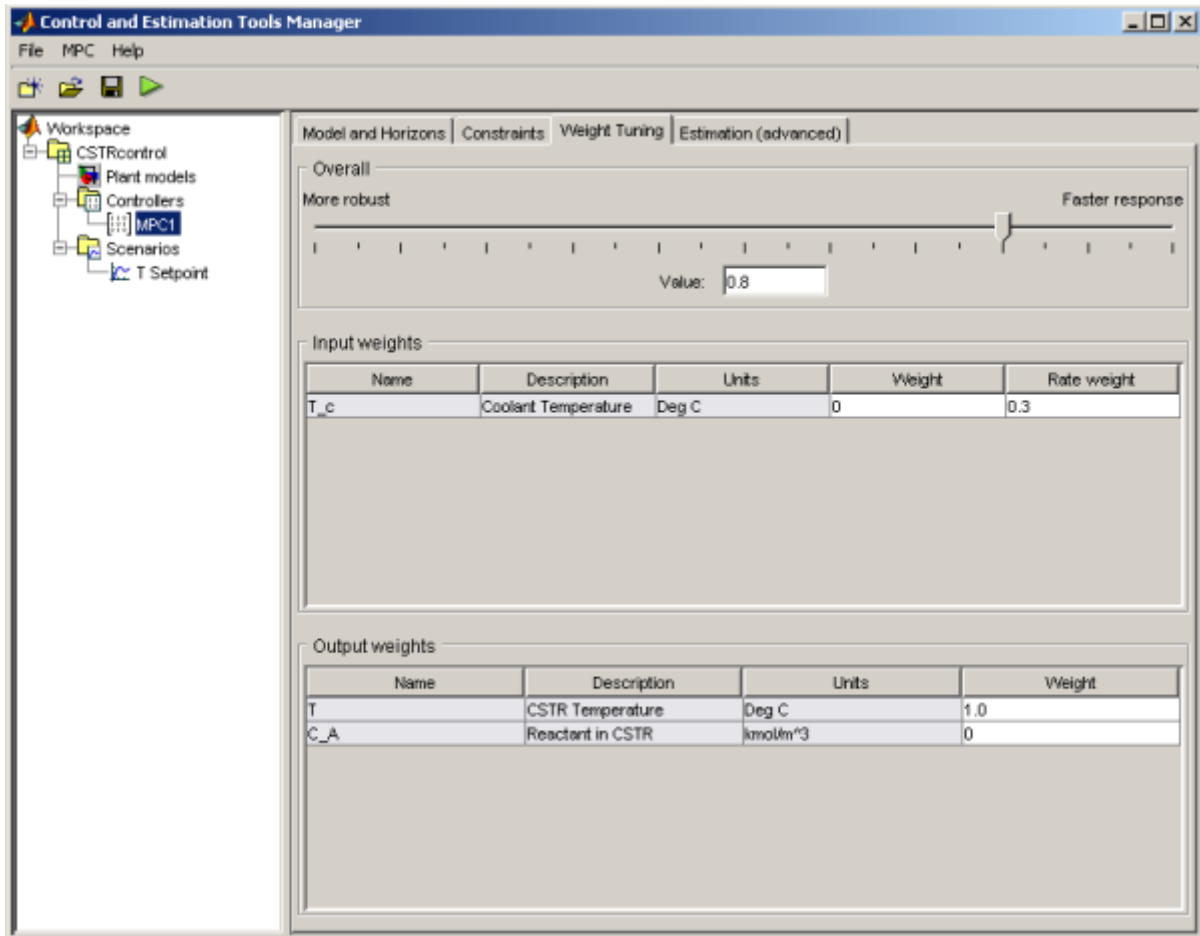
Leave all of the **Model and Horizons** tab settings at their default values for now.



**Figure 3-11: Controller Options – Models and Horizons Tab**

## Weight Tuning

Select the **Weight Tuning** tab. The view shown in Figure 3-12 appears.



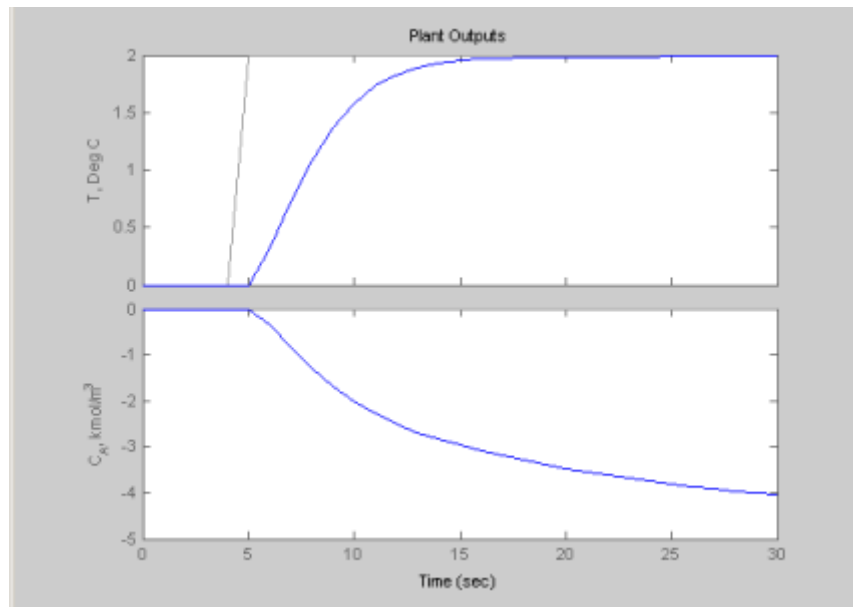
**Figure 3-12: Controller Options – Weight Tuning Tab**

Make the following changes:

- In the **Input weights** table, change the coolant temperature's **Rate weight** from the default 0.1 to 0.3.

- In the **Output weights** section, change the reactant concentration's **Weight** (last entry in second row) from the default 1.0 to 0.

Test these changes using the **T Setpoint** scenario (click the toolbar's Simulation button). Figure 3-13 shows that the CSTR temperature now tracks the setpoint change smoothly, reaching the new value in about 10 time units with no overshoot.



**Figure 3-13: Improved Setpoint Tracking for CSTR Temperature**

On the other hand, the reactant concentration,  $C_A$ , exhibits a larger deviation from its setpoint, which is being held constant at zero (compare to Figure 3-8, where the final deviation is about a factor of 4 smaller).

This behavior reflects an unavoidable trade-off. The controller has only one adjustment at its disposal: the coolant temperature. Therefore, it can't satisfy setpoints on both outputs.

### Output Weights

The output weights let you dictate the accuracy with which each output must track its setpoint. Specifically, the controller predicts deviations for each output over the prediction horizon. It multiplies each deviation by the output's weight value, and then computes the weighted sum of squared deviations,  $S_y(k)$ , as follows

$$S_y(k) = \sum_{i=1}^P \sum_{j=1}^{n_y} \left\{ w_j^y [r_j(k+i) - y_j(k+i)] \right\}^2$$

where  $k$  is the current sampling interval,  $k+i$  is a future sampling interval (within the prediction horizon),  $P$  is the number of control intervals in the prediction horizon,  $n_y$  is the number of plant outputs,  $w_j^y$  is the weight for output  $j$ , and the term  $[r_j(k+i) - y_j(k+i)]$  is a predicted deviation for output  $j$  at interval  $k+i$ .

The weights must be zero or positive. If a particular weight is large, deviations for that output dominate  $S_y(k)$ . One of the controller's objectives is to *minimize*  $S_y(k)$ . Thus, a large weight on a particular output causes the controller to minimize deviations in that output (relative to outputs having smaller weights).

For example, the default values used to produce Figure 3-8 specify equal weights on each output, so the controller is trying to eliminate deviations in both, which is impossible. On the other hand, the design of Figure 3-13 uses a weight of zero on the second output, so it is able to eliminate deviations in the first output.

---

**Note** The second output is unmeasured. Its predictions rely on the plant model and the temperature measurements. If the model were reliable, we could hold the predicted concentration at a setpoint, allowing deviations in the reactor temperature instead. In practice, it would be more common to control the temperature as done here, using the predicted reactant concentration as an auxiliary indicator.

---

You might expect equal output weights to result in equal output deviations at steady state. Figure 3-8 shows that this is not the case. The reason is that the controller is trying to satisfy several additional objectives simultaneously.

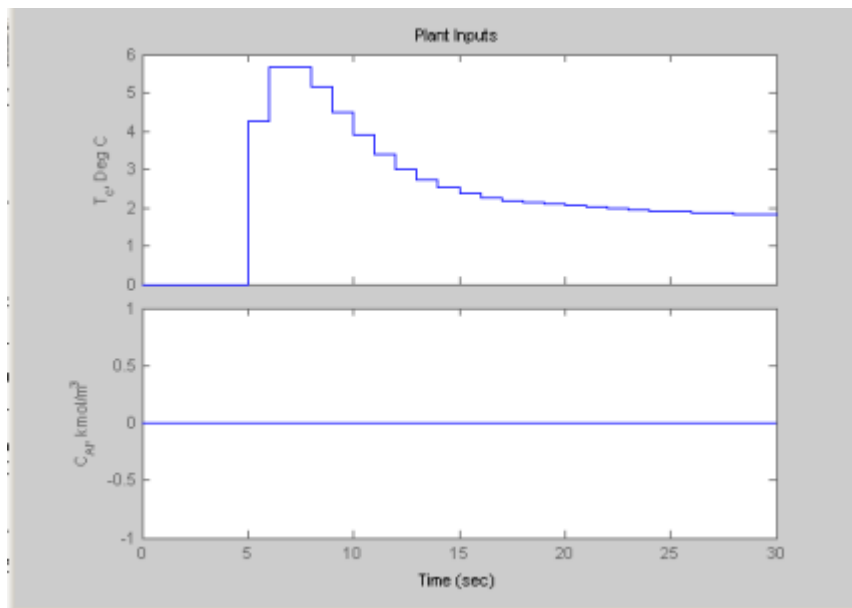


### Rate Weights

One such simultaneous objective is to minimize the weighted sum of controller adjustments, calculated according to

$$S_{\Delta u}(k) = \sum_{i=1}^M \sum_{j=1}^{n_{mv}} \left\{ w_j^{\Delta u} \Delta u_j(k+i-1) \right\}^2$$

where  $M$  is the number of intervals in the control horizon,  $n_{mv}$  is the number of manipulated variables,  $\Delta u_j(k+i-1)$  is the predicted adjustment in manipulated variable  $j$  at future (or current) sampling interval  $k+i-1$ , and  $w_j^{\Delta u}$  is the weight on this adjustment, called the *rate weight* because it penalizes the incremental change rather than the cumulative value. Increasing this weight forces the controller to make smaller, more cautious adjustments.



**Figure 3-14: Plant Inputs for Modified Rate Weight**

Figure 3-14 shows that increasing the rate weight from 0.1 to 0.3 decreases the move sizes significantly, especially the initial move (compare to Figure 3-9).

Setting the rate weight to 0.1 yields a faster approach to the  $T$  setpoint with a small overshoot, but the initial  $T_c$  move is about six times larger than needed to achieve the new steady state, which would be unacceptable in most applications (not shown; try it).

---

**Note** The controller minimizes the sum  $S_y(k) + S_{\Delta u}(k)$ . Changes in the coolant temperature have unequal effects on the two outputs, so the steady-state output deviations won't be equal, even when both output weights are unity as in Figure 3-8.

---

### Input Weights

The controller also minimizes the weighted sum of manipulated variable deviations from their nominal values, computed according to

$$S_u(k) = \sum_{i=1}^M \sum_{j=1}^{n_{mv}} \left\{ w_j^u [u_j(k+i-1) - \bar{u}_j] \right\}^2$$

where  $w_j^u$  is the *input weight* and  $\bar{u}_j$  is the nominal value for input  $j$ . In the above simulations, you used the default,  $w_j^u = 0$ . This is the usual choice.

When a sustained disturbance or setpoint change occurs, the manipulated variable must deviate permanently from its nominal value (as shown in Figure 3-9 and Figure 3-14). Using a nonzero input weight forces the corresponding input back toward its nominal value. Test this by running a simulation in which you set the input weight to 1. The final  $T_c$  value is closer to its nominal value, but this causes  $T$  to deviate from the new setpoint (not shown).

---

**Note** Some applications involve more manipulated variables than plant outputs. In such cases, it is common to define nonzero input weights on certain manipulated variables in order to hold them near their most economical values. The remaining manipulated variables eliminate steady-state error in the plant outputs.

---

## Blocking

The section “Weight Tuning” on page 3-22 used *penalty weights* to shape the controller’s response. This section covers the following topics:

- An alternative to penalty weighting, called *blocking*
- Side-by-side controller comparisons

To begin, select **Controllers** in the tree, and click the **New** button, creating a controller initialized to the Model Predictive Control Toolbox default settings. Rename this controller **Blocking 1** by editing the appropriate table cell.

Select **Blocking 1** in the tree, select its **Weight Tuning** tab, and set the **Weight** for output C\_A to 0 (see “Weight Tuning” on page 3-22 to review the reason for this). Leave other weights at their defaults.

Now select the **Model and Horizons** tab, and select its **Blocking** check box. This activates the blocking options. It also deactivates the **Control Horizon** option (the blocking options override it).

Set **Number of moves computed per step** to 2. Verify that **Blocking allocation within prediction horizon** is set to Beginning, the default.

Select **Controllers** in the tree, and use its **Copy** button to create two controllers based on **Blocking 1**. Rename these **Blocking 2** and **Blocking 3**. Edit their blocking options, setting **Blocking allocation within prediction horizon** to Uniform for **Blocking 2**, and to End for **Blocking 3**.

Select **Scenarios** in the tree. Rename the **T Setpoint** scenario to **T Setpoint 1**, and set its **Controller** option to **Blocking 1**.

Create two copies of **T Setpoint 1**, naming them **T Setpoint 2** and **T Setpoint 3**. Set their **Controller** options to **Blocking 2** and **Blocking 3**,

respectively. Now you should have three scenarios, identical except for the controller being used.

Delete the **MPC1** controller and select **T Setpoint 1** in the tree. Your view should resemble Figure 3-15, with three controllers and three scenarios in the tree.

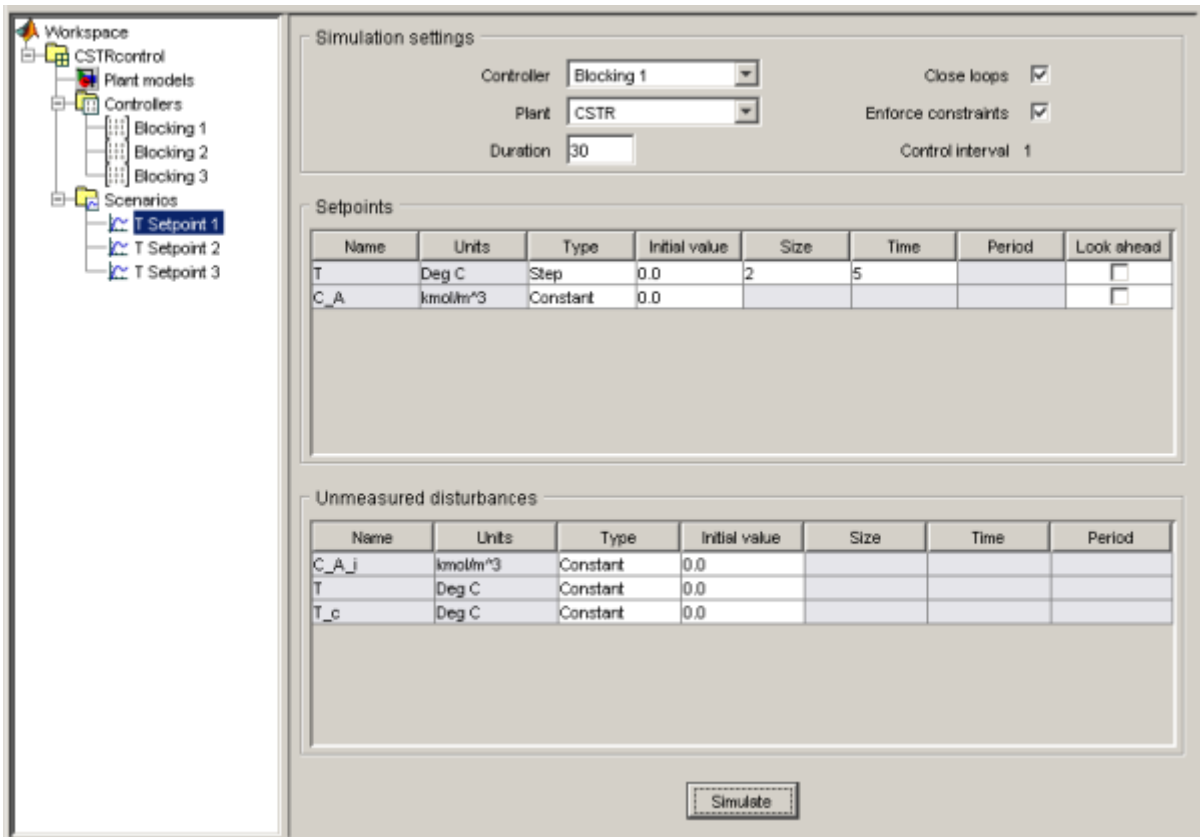
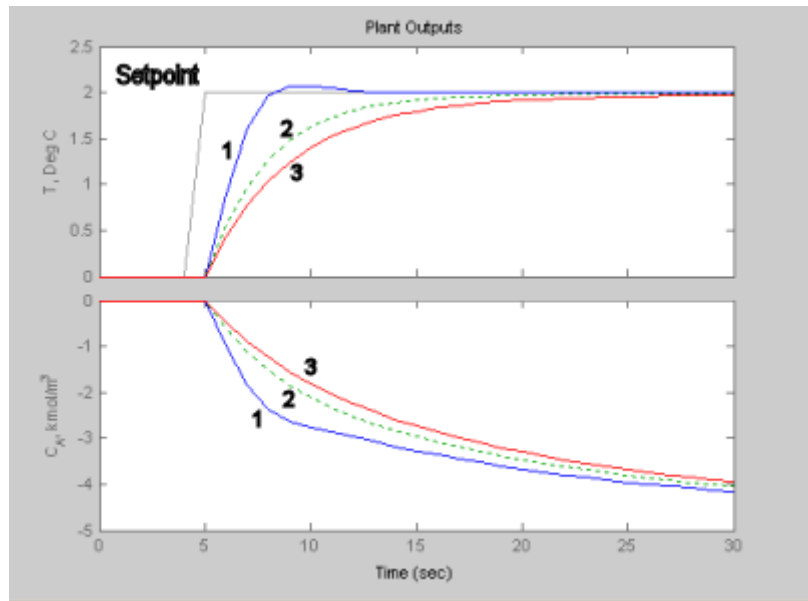


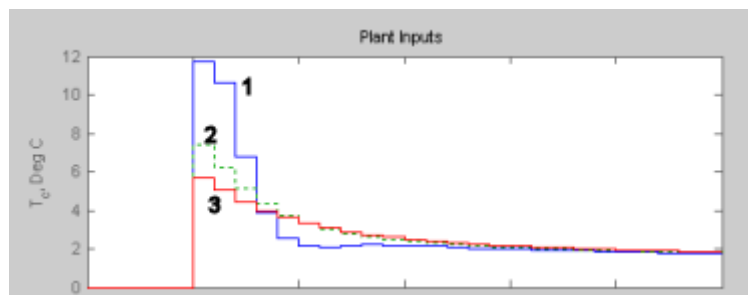
Figure 3-15: T Setpoint 1 Scenario

If any simulation plot windows are open, close them. This forces subsequent simulations to generate new plots.

Simulate each of the three scenarios. When you run the first, new plot windows open. Leave them open when you run the other two scenarios so all three results appear together, as shown in Figure 3-16 and Figure 3-17.



**Figure 3-16: Blocking Comparison, Outputs**



**Figure 3-17: Blocking Comparison, Manipulated Variable**

The numeric annotations on these figures refer to the three scenarios. Recall that **T Setpoint 1** uses the default blocking, which usually results in faster setpoint tracking but larger manipulated variable moves. The blocking options in **T Setpoint 2** and **T Setpoint 3** reduce the move size but make setpoint tracking more sluggish.

Results for **T Setpoint 3** are very similar to those shown in Figure 3-13 and Figure 3-14, where a penalty *rate weight* reduced the move sizes. If rate weights and blocking achieve the same ends, why does the Model Predictive Control Toolbox provide both features? One difference not evident in this simple problem is that blocking applies to all the manipulated variables in your application, but each rate weight affects one only.

---

**Note** To obtain the dashed lines shown for **T Setpoint 2**, activate the plot window and select **Property Editor** from the **View** menu. The Property Editor appears at the bottom of the window. Then select the curve you want to edit. The Property Editor lets you change the line type, thickness, color, and symbol type. Select the axis labels to see additional options.

---

By default, the Model Predictive Control Toolbox plots each scenario on the same plot. If you recalculate a revised scenario, it replots that result but doesn't change any others.

If you don't want to see a particular scenario, right-click on the plot and use the **Responses** menu option to hide it. (You can also close the plot window and recalculate selected responses in a fresh window.)

### Defining Manipulated Variable Constraints

Physical devices have limited ranges and rates of change. For example, the CSTR model's coolant might be restricted to a 20 degree range (from -10 to 10) and its maximum rate of change might be  $\pm 4$  degrees per control interval. If these are true physical restrictions, it's good practice to include them in the controller design. Otherwise the controller might attempt an unrealistic adjustment.

To compare constrained and unconstrained performance for the CSTR example, select the **Blocking 1** controller in the tree, and rename it **Unconstrained**.

Select its **Model and Horizons** tab and turn off (clear) its **Blocking** option. Increase **Control horizon** to 3, and reduce **Control Interval** to 0.25.

Delete the **Blocking 2** and **Blocking 3** controllers. (Click **OK** to dismiss the resulting warning messages.)

Copy the **Unconstrained** controller. Name this copy **MVconstraints**, and select its **Constraints** tab. Then enter the manipulated variable constraints shown in Figure 3-18.

Name	Units	Minimum	Maximum	Max down rate	Max up rate
T_c	Deg C	-10	10	-4	4

**Figure 3-18: Entering CSTR Manipulated Variable Constraints**

If any simulation plot windows are open, close them (to force fresh plots).

Select the **T Setpoint 1** scenario. If necessary, set its **Controller** option to Unconstrained. Change **Duration** to 15, and simulate the scenario.

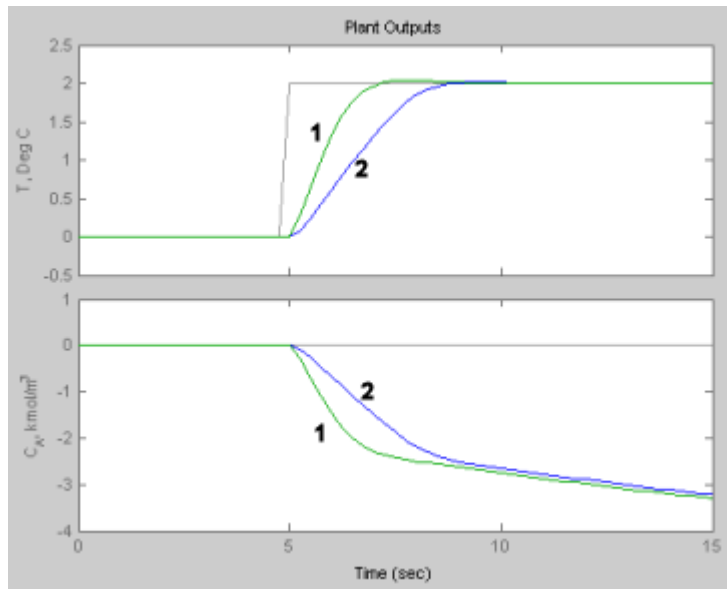
Select the **T Setpoint 2** scenario, set its **Controller** option to MVconstraints, change its **Duration** to 15, and simulate it. The results appear in Figure 3-19 and Figure 3-20.

The larger control horizon and smaller control interval cause the unconstrained controller to make larger moves (see Figure 3-20, curve 1). The output settles at the new setpoint in about 5 time units rather than the 10 required previously (compare curve 1 in Figure 3-19 to curve 1 in Figure 3-16).

Figure 3-20 (curve 2) shows that the **Max Up Rate** constraint limits the size of the first two moves to 4 degrees. The third move hits the **Maximum** constraint

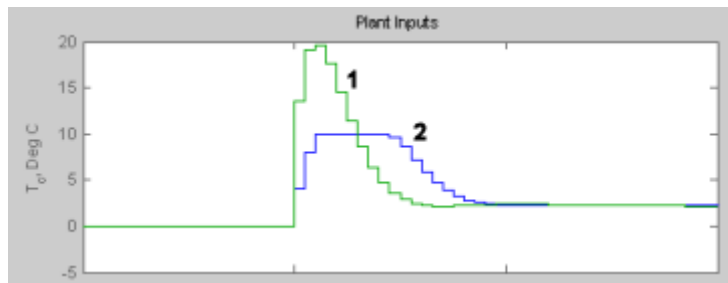
at 10 degrees. The coolant temperature remains saturated at its upper limit for the next 7 control intervals, then slowly moves back down to its final value.

Figure 3-19 (curve 2) shows that the output response is slower, but still settles at the new setpoint smoothly within about 5 time units. This demonstrates the *anti-windup* protection provided automatically by the Model Predictive Control Toolbox controller.



**Figure 3-19: CSTR Outputs, Unconstrained (1) and MVconstraints (2)**





**Figure 3-20: CSTR Manipulated Variable, Unconstrained (1) and MVconstraints (2)**

### Disturbance Modeling and Estimation

The previous sections tested the controller's response to setpoint changes. In process control, disturbance rejection is often more important. The Model Predictive Control Toolbox allows you to tailor the controller's disturbance response. The following example assumes that you have just completed the above section, and the design tool is still open.

Select the first controller in the tree and rename it **InputSteps**. (Its settings should be identical to the **Unconstrained** controller of the previous section.)

Copy this controller. Rename the copy **OutputSteps**. Select its **Estimation** tab. The initial view should be as in Figure 3-21. Note the following:

- The Model Predictive Control Toolbox default settings are being used. (The **Use Model Predictive Control Defaults** button restores these settings if you modify them.)
- The **Output disturbances** tab is selected, and the **Signal-by-signal** option is on. The graphic shows that the output disturbances add to each output.
- The tabular entries show, however, that these disturbance magnitudes are currently zero.

Model and Horizons | Constraints | Weight Tuning | Estimation (advanced)

Estimation parameters: MPC defaults Use MPC defaults

Overall estimator gain

Low gain High gain

Value:

Output disturbances | Input disturbances | Measurement noise

Signal-by-signal

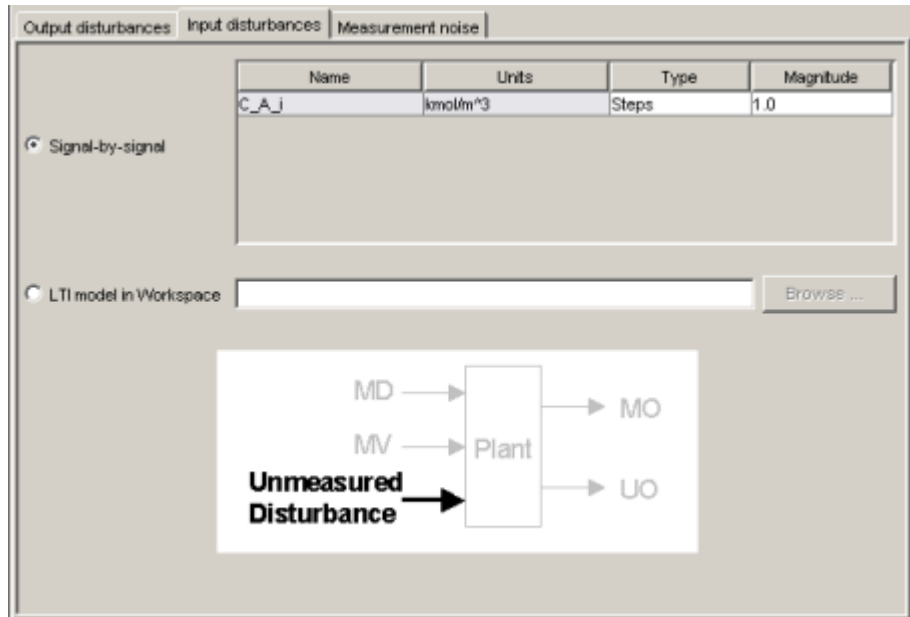
Name	Units	Type	Magnitude
T	Deg C	White	0.0
C_A	kmol/m <sup>3</sup>	White	0.0

LTI model in Workspace  Browse ...

The diagram shows a block labeled 'Plant' with three input arrows from the left labeled MD, MV, and UD. The output of the plant is a signal that enters a summing junction from the left, indicated by a '+' sign. A second input to the summing junction is an arrow pointing down from above, labeled 'Unmeasured Disturbance', also with a '+' sign. The output of the summing junction is labeled 'Outputs'.

**Figure 3-21: Default Output Disturbance Settings for CSTR**

Select the **Input disturbances** tab. (This would be inactive if the plant model had no unmeasured disturbances.) The view should change to that shown in Figure 3-22.



**Figure 3-22: Default Input Disturbance Settings for CSTR**

In this case the disturbance magnitude is nonzero, and the disturbance type is Steps. Thus, the controller assumes that disturbances enter as random steps (integrated white noise) at the plant's unmeasured disturbance input.

Select the **Measurement noise** tab, verifying that the controller is assuming white noise, magnitude 1 (not shown).

The following summarizes the Model Predictive Control Toolbox default disturbance modeling assumptions for this case:

- Additive output disturbances: none
- Unmeasured input disturbances: random steps (integrated white noise)
- Measurement noise: white

In general, if your plant model includes unmeasured disturbance inputs, the Model Predictive Control Toolbox default strategy will assume that they are dominant and sustained, as in the above example. This forces the controller to include an integrating mode, intended to eliminate steady-state error.

If the plant model contains no unmeasured input disturbances, the Model Predictive Control Toolbox assumes sustained (integrated white noise) disturbances at the measured plant outputs.

If there are more measured outputs than unmeasured input disturbances, it assumes sustained disturbances in both locations according to an algorithm described in the Model Predictive Control Toolbox online documentation.

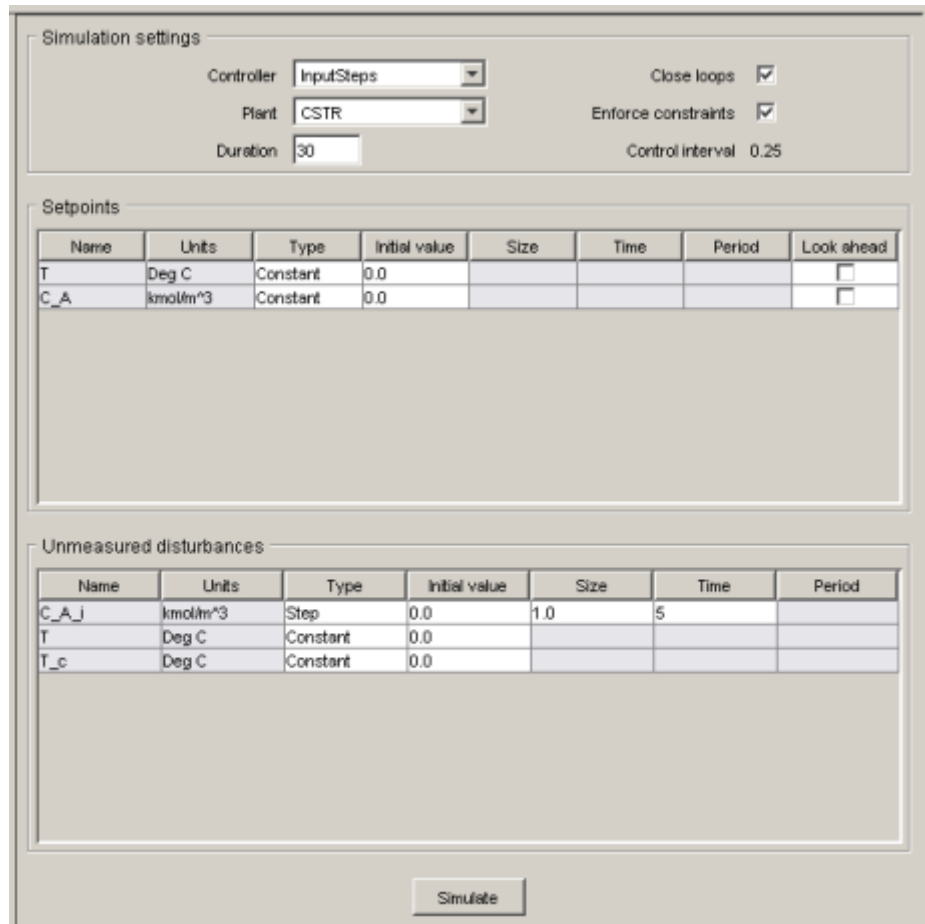
In any case, the design tool displays the assumptions being used.

To modify the estimation strategy in the OutputSteps controller, do the following:

- Select the **Input disturbances** tab. Set the disturbance **Type** to White, and its **Magnitude** to 0.
- Select the **Output disturbances** tab. For the T output, set the disturbance **Type** to Steps, and its magnitude to 1.

This *reverses* the default assumptions, i.e., the **OutputSteps** controller assumes that sustained disturbances enter at the plant output rather than at the unmeasured disturbance input. The **InputSteps** controller is still using the original (default) assumptions.

Next, select the first scenario in the tree. Rename it **Disturbance 1**, set its **Duration** to 30, define all setpoints as constant zero values, and define a unit-step disturbance in the unmeasured input, C\_A\_i. If necessary, set its **Controller** option to InputSteps. Figure 3-23 shows the final **Disturbance 1** scenario.



**Figure 3-23: CSTR Disturbance 1 Scenario**

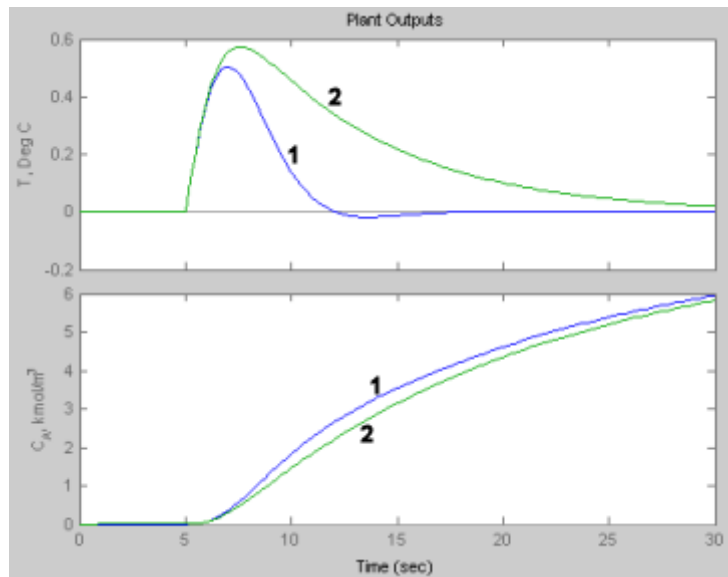
Copy **Disturbance 1**. Rename the copy **Disturbance 2**, and set its **Controller** option to OutputSteps.

If necessary, close any open simulation plot windows. Simulate both scenarios. Figure 3-24 and Figure 3-25 show the results.

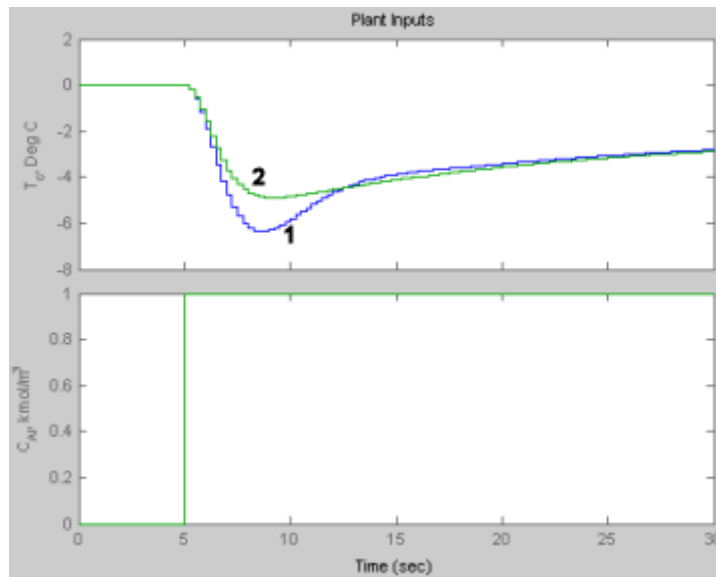
Figure 3-24 shows that default controller (case 1) returns to the setpoint in less than one third the time required by the modified controller (case 2). Its maximum deviation from the setpoint is also 10% smaller. Figure 3-25 shows

that in both cases the input moves are smooth and of reasonable magnitude. (It also shows the input disturbance.)

The default controller expects unmeasured disturbances to enter as defined in the scenarios, so it's not surprising that the default controller performs better. The main message here is that the difference can be large, so it merits design consideration.



**Figure 3-24: CSTR Outputs for Disturbance Scenarios 1 and 2**



**Figure 3-25: CSTR Inputs for Disturbance Scenarios 1 and 2**

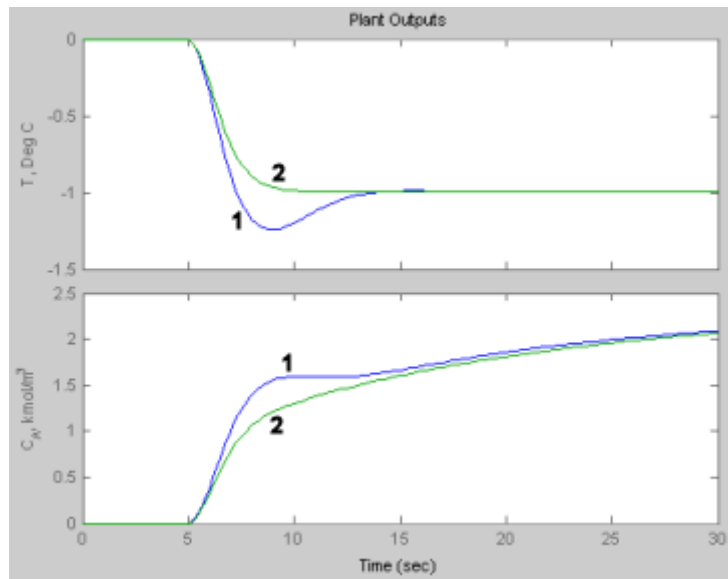
For comparison, reset the two scenarios so that the only disturbance is a one-degree step increase added to the measured reactor temperature. The modified controller (case 2) is designed for such disturbances, and Figure 3-26 shows that it performs better, but the difference is less dramatic than in the previous scenario. The default controller is likely to be best if the real process has multiple dominant disturbance sources.

---

**Note** In this modified scenario, the disturbance biases the measured temperature by +1 degree. The controller drives the biased measurement to the setpoint, so the true temperature exhibits a -1 degree steady-state error, as shown in Figure 3-26.

---





**Figure 3-26: CSTR Outputs, Output Disturbance Scenarios 1 and 2**

## Defining Soft Output Constraints

The discussion in “Weight Tuning” on page 3-22, defined temperature control as the primary goal for the CSTR application. The predicted (but unmeasured) reactant concentration,  $C_A$ , could vary freely.

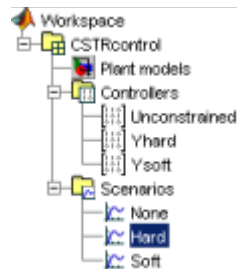
Suppose this were acceptable provided that  $C_A$  stayed below a specified maximum (above which unwanted reactions would occur). You can use an *output constraint* to enforce this specification.

Start with a single controller identical to the **InputSteps** controller described in “Disturbance Modeling and Estimation” on page 3-34. Rename it **Unconstrained**.

Right-click on **Unconstrained** in the tree and select **Copy**. Rename the copy **Yhard**. Make another copy, naming it **Ysoft**.

Similarly, start with a single scenario identical to Figure 3-23, except that its **Controller** setting should be Unconstrained. Name this scenario **None**.

Right-click on **None** in the tree and select **Copy**. Rename the copy **Hard**, and change its **Controller** setting to Yhard. Make another copy, naming it **Soft** and changing its **Controller** setting to Ysoft. Your tree should be as shown below.



Select your **Yhard** controller. On the **Constraints** tab, set the maximum for  $C_A$  to 3 as shown below.

Name	Units	Minimum	Maximum
T	Deg C		
$C_A$	kmol/m <sup>3</sup>		3

Click the **Constraint Softening** button to obtain the dialog shown in Figure 3-27. The **Input constraints** section shows the bounds on the inputs

and their rates, and *relaxation bands*, which let you *soften* these constraints. By default, input constraints are *hard*, meaning that the controller tries to prevent any violation.

**Specify relaxation bands**

Input constraints

Name	Units	Minimum	Min band	Maximum	Max band	Max down rate	Max down band	Max up rate	Max up band
T_c	Deg C	-10		10		-4		4	

Output constraints

Name	Units	Minimum	Min Band	Maximum	Max Band
T	Deg C				
C_A	kmol/m <sup>3</sup>			3	0

Overall constraint softness

Soft constraints Hard constraints

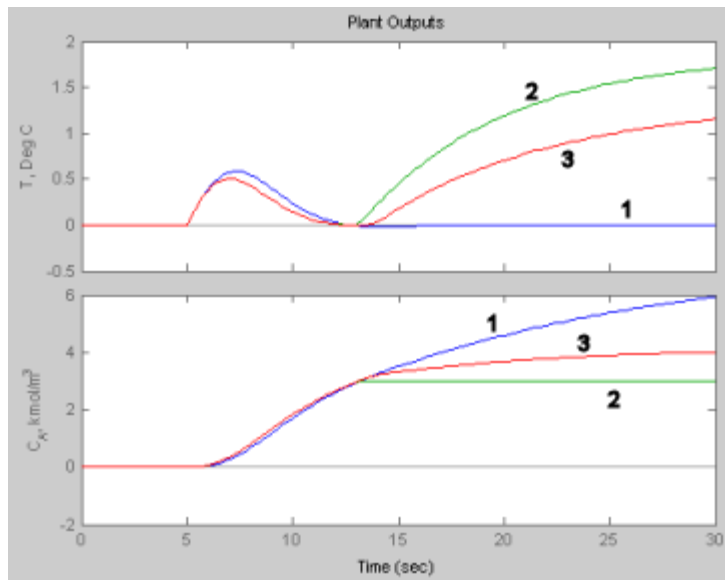
Value:

**Figure 3-27: Constraint Softening Dialog**

The **Output constraints** section lists the output limits and their relaxation bands. By default, the output constraints are *soft*. Make the  $C_A$  upper limit hard by entering a zero as its relaxation band (as in Figure 3-27).

Select the **Ysoft** controller. Define a soft upper bound on  $C_A$  by using the same settings shown in Figure 3-27, but with a relaxation band of 100 instead of 0.

Simulate the three scenarios in the order they appear in the tree, i.e., **None**, **Hard**, **Soft**. The resulting output responses appear in Figure 3-28.



**Figure 3-28: Constraint Softening Scenarios: 1 = None, 2 = Hard, 3 = Soft**

Curve 1 is without output constraints, which is identical to curve 1 in Figure 3-24. This controller allows the  $C_A$  output to vary freely, but the controlled temperature returns to its setpoint within 10 time units after the disturbance happens.

Curve 2 shows the behavior with a hard upper limit at  $C_A = 3$ . Once  $C_A$  reaches this bound, the controller must use its one manipulated variable ( $T_c$ ) to satisfy the constraint, so it's no longer able to control the temperature.

Curve 3 shows the result for a soft constraint. The controller reaches a compromise between the competing objectives: temperature control and constraint satisfaction. As you'd expect, performance lies between the curve 1 and curve 2 extremes.

The numerical value of the relaxation band represents a *relative tolerance* for constraint violations, not a strict limit (if it were the latter, it would be a hard constraint). If you were to increase its relaxation band (currently set at 100), performance would move toward Case 1, and vice versa.

If you have multiple constraints, you can harden or soften them simultaneously using the slider at the bottom of the controller's constraint softening dialog (see Figure 3-27).

In general, you'll have to experiment to determine the settings that provide appropriate trade-offs for your application. In particular, the relaxation band settings interact with those on the controller's **Weight Tuning** tab (see "Weight Tuning" on page 3-22).

Another important factor is the expected numerical range for each variable. For example, if a particular variable stays within  $\pm 0.1$  of its nominal value, it should have a small relaxation band relative to another variable having a range of  $\pm 100$ .

For details on the Model Predictive Control Toolbox constraint softening formulation, see the "Optimization Problem" section in the Model Predictive Control Toolbox online documentation.

## Robustness Testing

It's good practice to test your controller's sensitivity to prediction errors. Classical phase and gain margins are one way to quantify robustness for a SISO application. The Robust Control Toolbox and  $\mu$ -Analysis and Synthesis Toolbox provide sophisticated approaches for MIMO systems. It can also be helpful to run simulations. The following example illustrates the simulation approach.

### Plant Model Perturbation

Use the following code to create a perturbed version of the CSTR model:

```
CSTRp = CSTR;  
CSTRp.a=[ -0.0303   -0.0113  
          -0.0569   -0.1836];  
CSTRp.b=[ -0.0857    0.0191  
          0.1393    0.4241];
```

This creates a copy of CSTR called CSTRp, then replaces the state space  $A$  and  $B$  matrices with perturbed versions (compare to the originals defined in “State-Space Format” on page 2-5). Use the following command to compare the two step responses:

```
step(CSTR, CSTRp)
```

Observe the difference in the responses (not shown).

Select **Plant models** in the tree. Click the **Import** button and import the CSTRp model.

### Simulation Tests

Delete all controllers except the first one in the tree. If necessary, make its settings identical to **Unconstrained** (see “Defining Manipulated Variable Constraints” on page 3-30).

Delete all scenarios except the first, naming that **Accurate Model**. Define its properties as shown in Figure 3-29. The scenario begins with a step change in the temperature setpoint, followed 25 time units later by a step disturbance in the reactant entering the CSTR.

Copy **Accurate Model**. Rename the copy **Perturbed Model**, and set its **Plant** option to CSTRp. Thus, both scenarios use the same controller, which is based on the CSTR model, but the **Perturbed Model** scenario uses a different model to represent the “real” plant. This tests the controller’s robustness to a change in plant parameters.

Simulation settings

Controller: Unconstrained      Close loops:

Plant: CSTR      Enforce constraints:

Duration: 50      Control interval: 0.25

Setpoints

Name	Units	Type	Initial value	Size	Time	Period	Look ahead
T	Deg C	Step	0.0	1.0	1.0		<input type="checkbox"/>
C_A	kmol/m <sup>3</sup>	Constant	0.0				<input type="checkbox"/>

Unmeasured disturbances

Name	Units	Type	Initial value	Size	Time	Period
C_A_i	kmol/m <sup>3</sup>	Step	0.0	1.0	25	
T	Deg C	Constant	0.0			
T_c	Deg C	Constant	0.0			

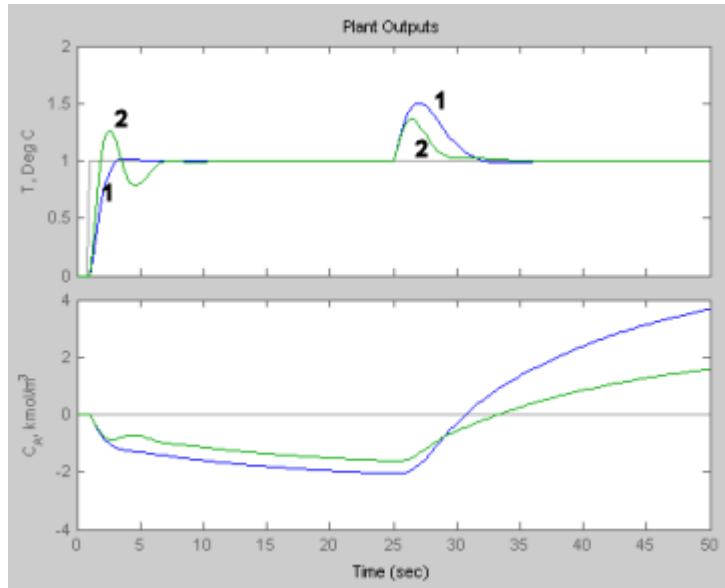
Simulate

**Figure 3-29: Robustness Test, Accurate Plant Model Scenario**

Simulate the two scenarios. Figure 3-30 shows the output responses. As expected, setpoint tracking degrades when the model is inaccurate, but performance is still acceptable.

The disturbance rejection appears to *improve* with the perturbed model. This is a consequence of the perturbations used. The gain for the  $T/C_{Ai}$  output/input

pair is about 15% smaller in the CSTRp model, which has two beneficial effects: the actual impact of the disturbance is reduced, and the controller is aggressive because it expects a larger impact.



**Figure 3-30: Robustness Test, Accurate Model (1) and Perturbed Model (2)**

---

**Note** MIMO applications are usually more sensitive to model error than SISO applications, so robustness testing is especially recommended for MIMO cases.

---



## Plant Models with Delays

Unlike many controller design approaches, the Model Predictive Control Toolbox can deal directly with models that include delays. A typical example is the distillation column model, DC, introduced in “Multiinput Multioutput (MIMO) Plants” on page 2-11. It includes a delay in each input/output channel.

You don’t need to approximate the delays in order to use this model in the Model Predictive Control Toolbox. On the other hand, the presence of delays will influence controller performance, and your controller specifications should account for them. The following example provides some guidelines and covers the following topics:

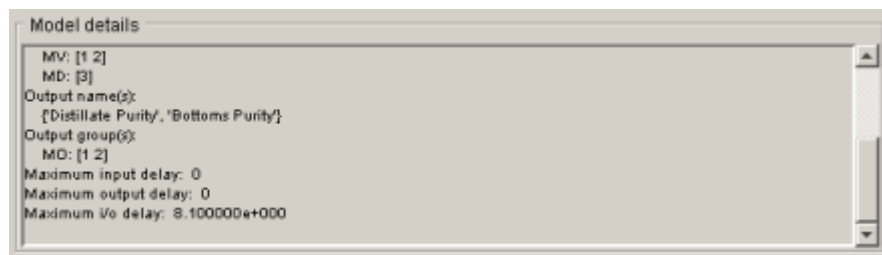
- “Importing the Plant Model” on page 3-49
- “Specifying Controller Horizons” on page 3-50

### Importing the Plant Model

Before continuing, the DC model must be in your workspace. (See “Multiinput Multioutput (MIMO) Plants” on page 2-11 for the commands needed to create it.)

Start the Model Predictive Control Toolbox design tool (type `mpctool` at the MATLAB command line). Import the DC model (see “Loading a Plant Model” on page 3-3).

Select **Plant models** in the tree. The DC model should be the only one listed. Scroll the **Model details** view to show the last few lines, which should appear as in Figure 3-31.



**Figure 3-31: DC Model Details, Maximum Delay Values**

## Specifying Controller Horizons

The model's maximum I/O delay is 8.1 minutes. It is good practice to specify the prediction and control horizons such that

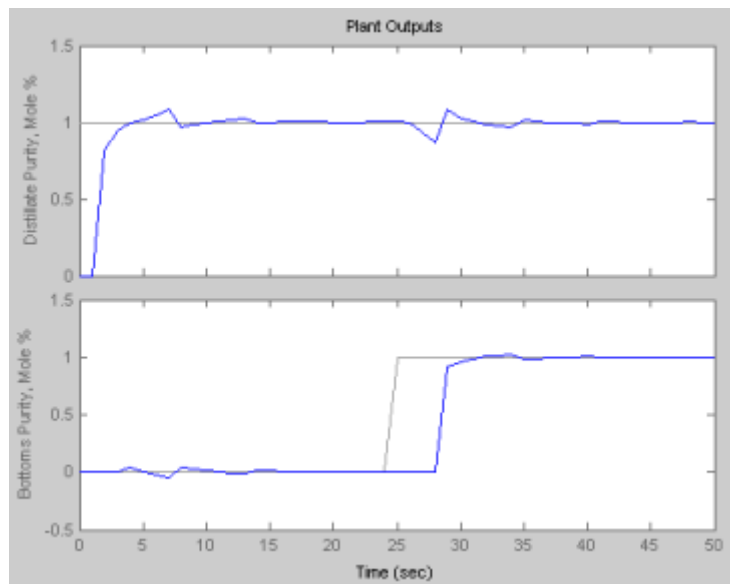
$$P - M \gg t_{d,max} / \Delta t$$

where  $P$  is the prediction horizon,  $M$  is the control horizon,  $t_{d,max}$  is the maximum delay, and  $\Delta t$  is the control interval.

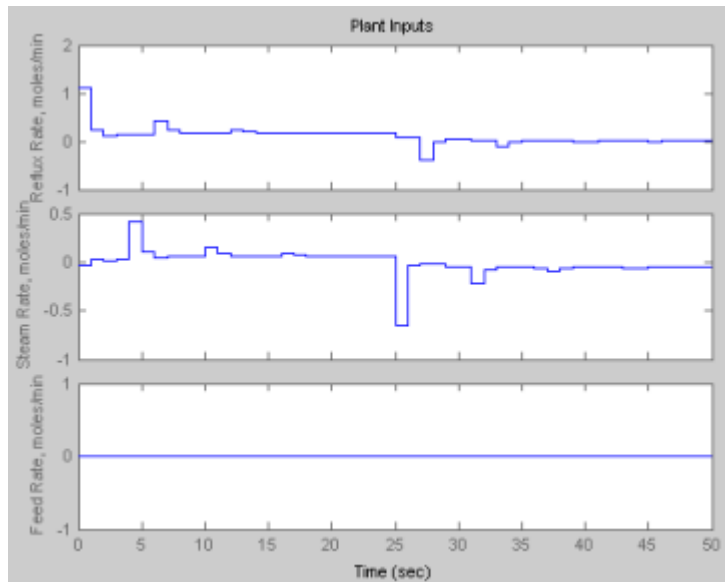
Select **MPC1** (the default controller name) in the tree. Select the **Model and Horizons** tab, and set **Control interval** to 1, a reasonable choice if the closed loop response time is to be of order 5-10 minutes.

Given the amount of plant delay and the specified control interval, the default horizons,  $P = 10$ ,  $M = 2$ , would be a poor choice. Instead, set **Prediction horizon** to 30, and **Control horizon** to 5.

Select **Scenario1** in the tree. Set **Duration** to 50. Define a constant setpoint of 1 for the first output (the distillate purity). Define a step increase of 1 in the second output's setpoint, occurring at  $t = 25$ . All other signals should be zero. Simulate the scenario. Figure 3-32 and Figure 3-33 show the results.



**Figure 3-32: DC Setpoint Response Scenario, Outputs**



**Figure 3-33: DC Setpoint Response Scenario, Inputs**

As seen in Figure 3-32, the first output cannot respond for a minimum of 1 minute, the delay in the  $y_1/u_1$  transfer function. After that, it reaches the setpoint in two minutes and settles quickly. Similarly,  $y_2$  cannot respond for a minimum of 3 minutes, the delay in the  $y_2/u_2$  transfer function, but settles rapidly thereafter. Changing one setpoint disturbs the other output, but the magnitude of this *interaction* is less than 10%.

Figure 3-33 shows that the initial input moves are more than 5 times the final change. Also, there are periodic pulses in the control action as the controller attempts to counteract the delayed effects of each input on the two outputs.

You can moderate these effects using the weights (see “Weight Tuning” on page 3-22). Instead, define a custom blocking strategy as illustrated in Figure 3-34. This uses five moves as before, but allocates them in increasing block sizes.

Figure 3-35 and Figure 3-36 show the corresponding simulation results. The initial input moves are much smaller, and the moves are less oscillatory overall. The trade-off is a slower output response with about 20% interaction.

Plant model: DC

Horizons

Control interval (time units): 1

Prediction horizon (intervals): 30

Control horizon (intervals): 5

Blocking (advanced)

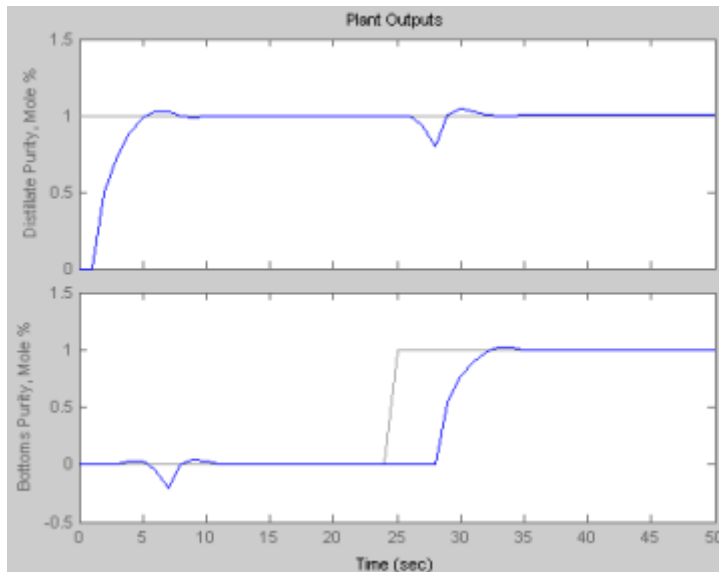
Blocking

Blocking allocation within prediction horizon: Custom

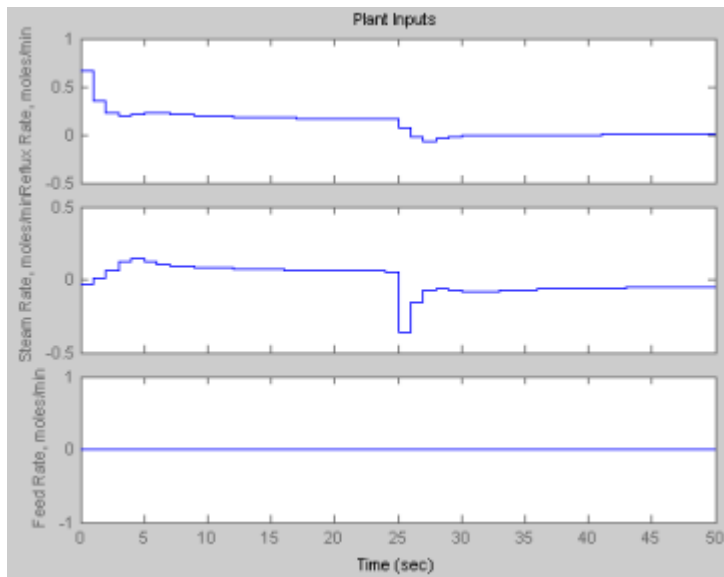
Number of moves computed per step: 5

Custom move allocation vector: [1 3 6 10 10]

**Figure 3-34: DC Model, Custom Blocking Strategy**



**Figure 3-35: Output Responses for Setpoint Scenario with Blocking**



**Figure 3-36: Input Moves for Setpoint Scenario with Blocking**

## Nonsquare Plants

A *nonsquare* plant has an unequal number of manipulated variables and output variables. This is common in practice, and the Model Predictive Control Toolbox supports an excess of manipulated variables or outputs. In such cases you will usually need to modify the default Model Predictive Control Toolbox settings.

This section covers the following topics:

- “More Outputs Than Manipulated Variables” on page 3-55
- “More Manipulated Variables Than Outputs” on page 3-56

### More Outputs Than Manipulated Variables

When there are excess outputs, it is impossible to hold all of them at setpoints. You have the following options:

- 1 Enforce setpoints on all outputs, in which case all will deviate from their setpoints to some extent.
- 2 Specify that certain outputs need not be held at setpoints by setting their weights to zero on the controller’s **Weight tuning** tab.

The initial test of the CSTR controller used option 1 (the default), which caused both outputs to deviate from their setpoints (see Figure 3-8). You can adjust the offset in each output by changing the output weights. Increasing an output weight decreases the offset in that output (at the expense of increased offset in other outputs).

The modified CSTR controller used option 2 (see the discussion in “Weight Tuning” on page 3-22). In general, if the application has  $N_e$  more outputs than manipulated variables, setting  $N_e$  output weights to zero should allow the remaining outputs to be held at setpoints (unless the manipulated variables are constrained). This was the case for the modified CSTR controller (see Figure 3-13).

Outputs that have been “sacrificed” by setting their weights to zero can still be useful. If measured, they can help the controller to estimate the plant’s state, thereby improving its predictions. They can also be used as indicators, or as variables to be held within an operating region defined by output constraints.

## More Manipulated Variables Than Outputs

In this situation, the default Model Predictive Control Toolbox settings should provide offset-free output-setpoint tracking, but the manipulated variables are likely to drift.

One way to avoid this is to use *manipulated variable setpoints*. If there are  $N_e$  excess manipulated variables and you hold  $N_e$  of them at target values, the rest should not drift. Rather, they will attain the values needed to eliminate output offset.

To define a manipulated variable setpoint, do the following:

- 1 Enter the setpoint value in the **Nominal** field in the signal properties view – see Figure 3-3
- 2 Assign a nonzero *input weight* using the **Weight** entry on the controller’s **Weight tuning** tab – see Figure 3-12

In step 2, the magnitude of the input weight determines the extent to which the manipulated variable can deviate from its target during a transient. See “Input Weights” on page 3-26 for more discussion and mathematical details.

You might want to allow such deviations temporarily in order to provide better output setpoint tracking. In that case, use a relatively small input weight. If you want the manipulated variable to stay near its target value at all times, increase its input weight.

Another way to avoid drift is to constrain one or more manipulated variables to a narrow operating region. You can even hold an MV constant by setting its lower and upper bounds to the same value (in which case its nominal value should also be set to this value), or by setting both of its rate constraints to zero. To define constraints, use the controller’s **Constraints** tab (see “Defining Manipulated Variable Constraints” on page 3-30).



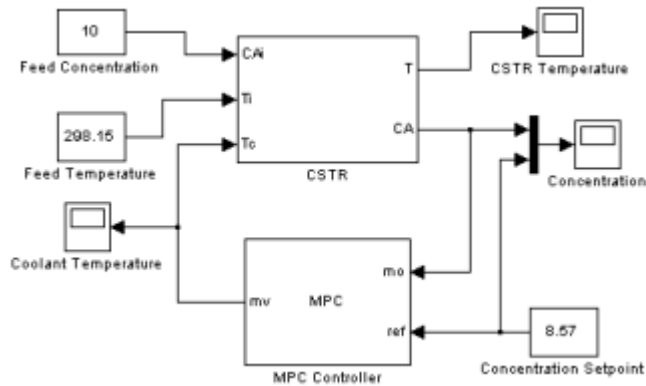
## Nonlinear Plants

The Model Predictive Control Toolbox provides a Simulink block you can use to control a plant modeled in Simulink. This section illustrates the procedure using the nonlinear CSTR model introduced earlier (see “Using Simulink to Develop LTI Models” on page 2-19). It covers the following topics:

- “The MPC Controller Block” on page 3-57
- “Initiating the Controller Design” on page 3-58
- “Validating the Linearized Model” on page 3-61
- “Modifying the Linearized Model” on page 3-63
- “Linear Simulation Tests” on page 3-64
- “Nonlinear Simulation Tests” on page 3-66
- “Modifying the Controller Using the Design Tool” on page 3-68
- “Exiting the Design Tool” on page 3-68

### The MPC Controller Block

Figure 3-37 shows CSTR with an added MPC Controller block. (The model is called `CSTR_MPC.mdl` in the Model Predictive Control Toolbox demos folder.) The intent is to hold the residual concentration at a setpoint by adjusting the coolant temperature. The feed concentration and temperature inputs are disturbances, and the reactor temperature is an auxiliary, uncontrolled output. The concentration has a setpoint (reference), currently fixed at 8.57.

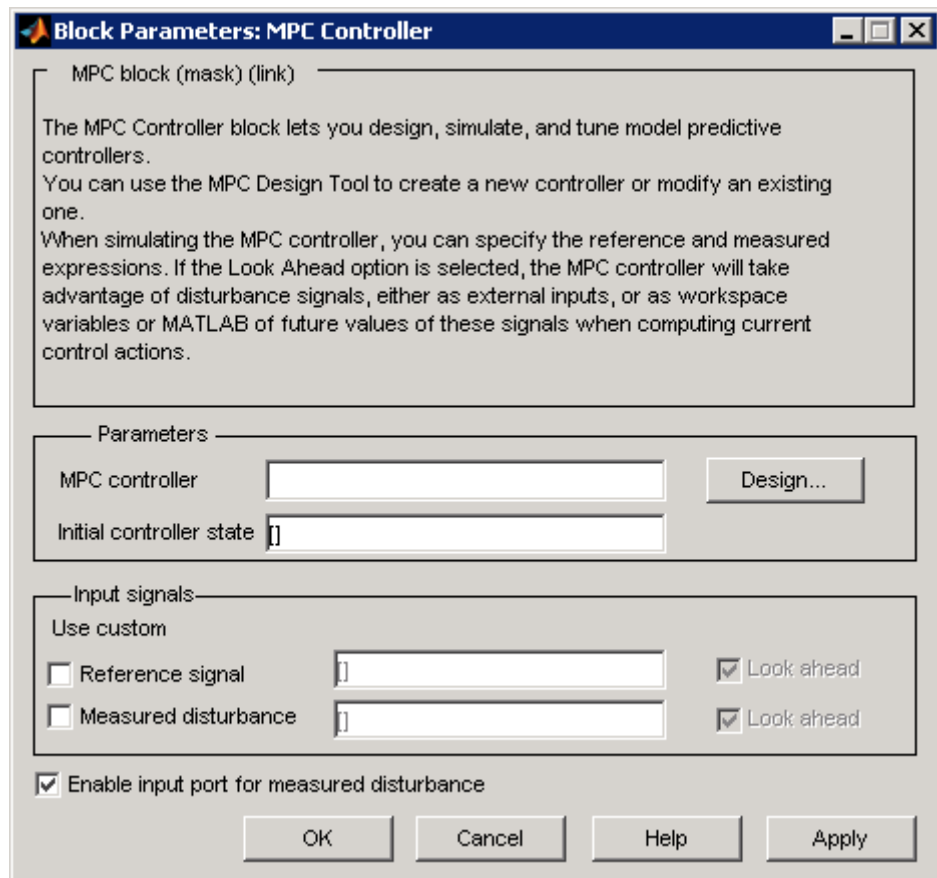


**Figure 3-37: Simulink Model for CSTR Control**

**Note** If you have the Model Predictive Control Toolbox and Simulink installed, the MPC Controller block appears in your Simulink block library. If necessary, see the Simulink documentation for details on how to use library blocks to construct a model.

## Initiating the Controller Design

Once you've installed an MPC Controller block in your Simulink model, as in Figure 3-37, you need to configure it. Click on it to open its *block mask*, shown below.

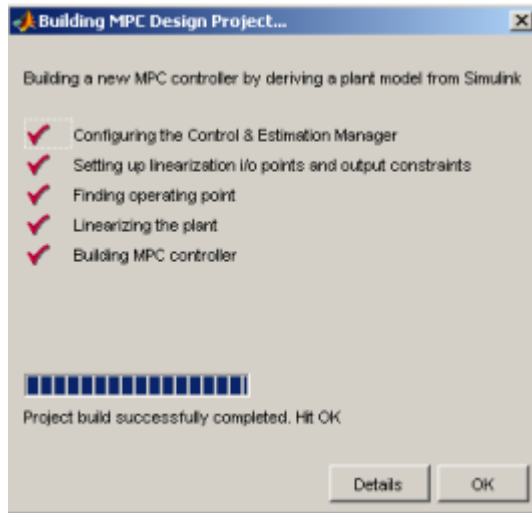


**Figure 3-38: MPC Controller Block Mask**

You *must* define the **MPC controller** parameter. All others are optional. If you had used the design tool to create a controller and had saved it in your workspace, you could type its name in the **MPC controller** box and you'd be ready to test it.

Instead, create the controller using the MPC Toolbox design tool. Click the **Design** button. A dialog (not shown) asks you to enter the number of *manipulated variables*, i.e., the plant inputs the controller can adjust to achieve its objectives. The coolant temperature is the only manipulated variable, so enter 1 and click **OK**.

The progress window shown below appears to mark the completion of each controller initialization step.



They are as follows:

- 1 Open the Control and Estimation Tools Manager, and install a *project* having the same name as the Simulink model (CSTR\_MPC in this case).
- 2 Install a *linearization task* within the project and determine the plant's I/O points, i.e., the signals connected to the MPC Controller block's *mv* (manipulated variable) and *mo* (measured output) ports.
- 3 Determine a default *operating point*. This requires assumptions that might be incorrect, as demonstrated in the next section. In particular, the default outputs are zero.
- 4 Calculate a *linearized plant model* at this operating point using the linearization tool in Simulink Control Design (see “Linearization Using Simulink Control Design” on page 2-19). The controller is open loop during this step, so it involves only the blocks between the plant inputs and outputs (as determined in step 2).

- 5 Install a *Model Predictive Control Toolbox design task* in the project, use the linearized model from step 4 to define a default controller (named **MPC1**), and enter its name as the block mask's **MPC controller** parameter.

When the last step has been completed, click **OK** to close the progress window.

## Validating the Linearized Model

Automatic linearization requires assumptions that might be incorrect. You should always validate a linearized model before using it in a controller.

As shown below, the controller task is the **MPC Controller** node, the name of the corresponding Simulink block.

The screenshot shows the Simulink workspace for a project named 'Project - CSTR\_MPC'. The workspace tree on the left is expanded to show the 'MPC Controller' node, which contains an 'Operating Point' block. The 'States' tab of the 'Operating Point' block is active, displaying the following data:

State Name	Desired Value	Actual Value	Desired dx	Actual dx
<b>CSTR_MPC/CSTR/Integrator</b>				
State - 1	[ 0 , Inf ]	229.3538	0	0.00059453
<b>CSTR_MPC/CSTR/Integrator1</b>				
State - 1	[ 0 , Inf ]	5	0	4.9991
<b>CSTR_MPC/MPC Controller/sfunction</b>				
State - 1	[ -Inf , Inf ]	-0.00014165	0	0

---

**Note** In general, a Simulink model can contain multiple controllers, in which case the project would include multiple controller design tasks, each with a unique name.

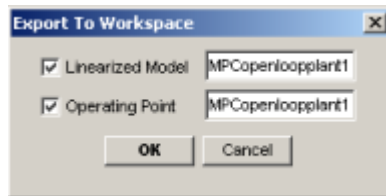
---

Start by examining the default operating point. Expand the tree until it appears as above, and then select **Operating Point**. The **States** tab should be active as shown (if not, click on it).

By default, the Simulink Control Design linearization tool tries to find a steady-state point, i.e., it sets the **Desired dx** column to zero. As shown above, the **Actual dx** column contains a value that is far from zero.

Next, click the **Outputs** tab. By default, the linearization tool sets the desired plant output to zero. In the CSTR, this would require 100% conversion of the reactant, which is impossible, and the **Outputs** tab confirms that the desired values were not achieved.

Finally, export the linearized model to your workspace. Right-click on the **MPC open loop plant 1** node and choose **Export** to obtain the dialog shown below.

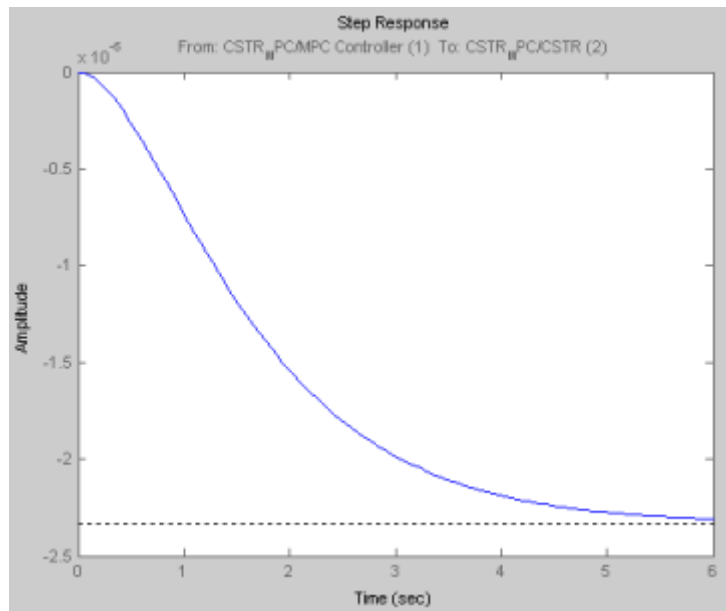


Rename **Linearized Model** to `Plant1`. Clear the **Operating Point** check box (because there's no reason to export it) and click **OK**. This exports the linearized model as an LTI object named `Plant1`.

At the MATLAB command line, type

```
step(Plant1)
```

You should obtain the following plot.



Thus, the linearized model predicts that a unit step *increase* in the coolant temperature will *decrease* the residual concentration, which is qualitatively correct. (Increasing the coolant temperature increases the reactor temperature. This increases the reaction rate, and the residual concentration decreases.)

The predicted magnitude is very small, however: of order  $10^{-6}$ . It should be of order  $10^{-2}$  (you can verify this by removing the controller block from the diagram and running an open-loop step-response simulation). The assumption of zero plant outputs used to generate the default operating point are the cause. If this incorrect model were used in the controller, the coolant-temperature adjustments would be far too large, probably resulting in unstable behavior.

## Modifying the Linearized Model

Therefore, the next step is to define a reasonable operating point and calculate a corresponding linearized model.

In the navigation tree, click the **MPC Controller** node. Notice that the nominal input and output signal values are zero by default. As discussed in “Linearization Using Simulink Control Design” on page 2-19, a coolant

temperature of 298.15 K is the steady state for the plant’s initial condition (with the residual concentration at 8.57). Thus, in the **Input signal properties** table, set the coolant temperature’s nominal value to 298.15, as shown below, and in the **Output signal properties** table, set the concentration’s nominal value to 8.57. Also assign more descriptive signal names, as shown.

Input signal properties				
Name	Type	Description	Units	Nominal
Coolant Temperature	Manipulated			298.15
Output signal properties				
Name	Type	Description	Units	Nominal
Concentration	Measured			8.57

Click the **Import Plant** button to obtain the **Plant Model Importer** dialog. Then do the following:

- 1** Select the **Linearized Plant from Simulink** tab.
- 2** Change the **Linearization model name** to Plant2.
- 3** Select **Create a new operating condition from MPC I/O values**, and then click **OK**. A new model node named **Plant2** and its operating point will appear within your design task.
- 4** Expand the **Controllers** node to expose the **MPC1** node and select it. On its **Model and Horizons** pane, set **Plant model** to Plant2. This replaces the default (invalid) model with the modified one.

Leave the other controller settings at their default values.

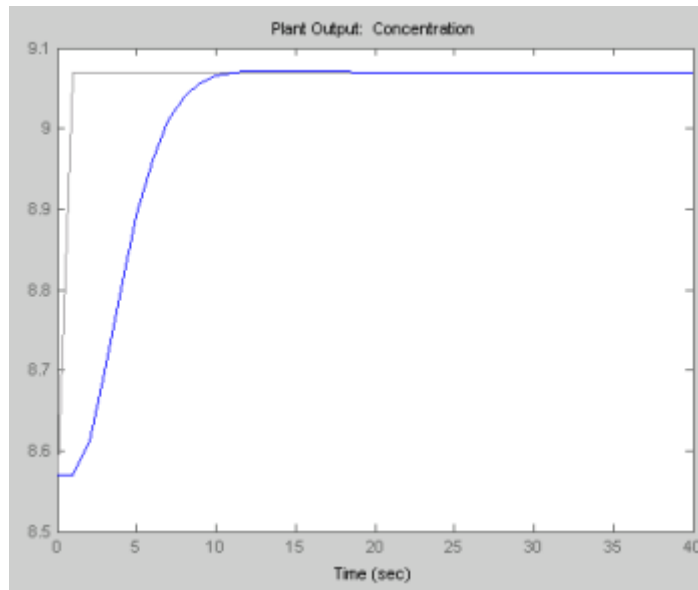
## Linear Simulation Tests

It’s good practice to test a controller in linear simulations before trying it on the nonlinear plant. Expand the **Scenarios** node and select **Scenario1**. Set **Plant** to Plant2, and set **Duration** to 40. Finally, define a step increase of 0.5 units in the concentration setpoint starting at time  $t = 1$ , as shown below.



Setpoints					
Name	Units	Type	Initial value	Size	Time
Concentration		Step	8.57	0.5	1

Click the scenario's **Simulate** button. This linear test predicts a smooth, rapid approach to the new setpoint with minimal overshoot, as shown below.



The corresponding coolant temperature adjustments are reasonable (not shown). If anything, you might want to make the controller more aggressive by adjusting its tuning weights (see “Weight Tuning” on page 3-22).

---

**Note** This plant’s steady-state gain is of order 0.01. Therefore, the default tuning weights lead to a relatively sluggish response. In general, you must adjust the tuning weights to compensate for the plant’s natural input/output response magnitudes.

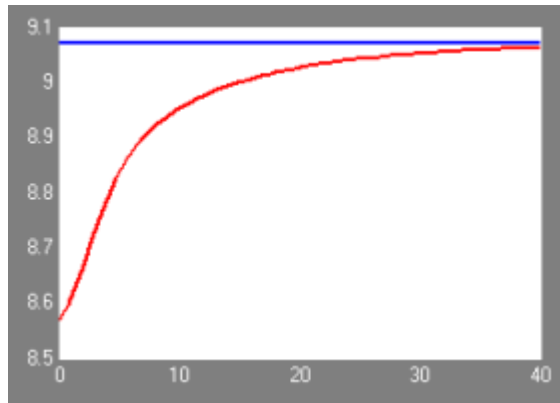
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You can also run tests to verify that the controller responds rapidly to either of the two unmeasured disturbances (not shown).

## Nonlinear Simulation Tests

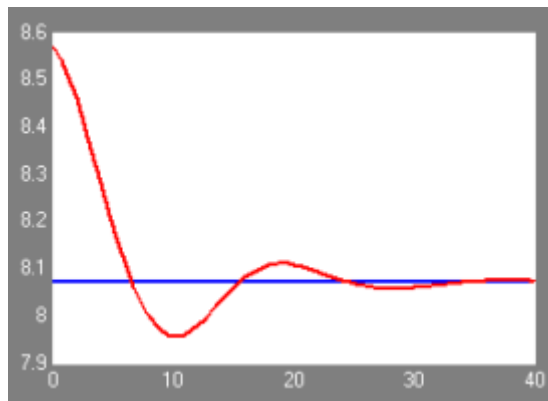
The controller seems to be performing well in linear tests, so try it with the nonlinear plant. As will be demonstrated below, the design tool makes it easy to run a simulation, change the controller design if performance is unsatisfactory, and retest.

In the CSTR\_MPC model window, change the concentration setpoint to 9.07. This simulates a 0.5-unit step increase in the setpoint at time  $t = 0$ . Run the Simulink simulation. The MPC Controller block automatically obtains the most recent **MPC1** definition from the design tool. If necessary, open the model's concentration scope block. The result should be as shown below.



The nonlinear response is more sluggish than the linear prediction. The controller reduces the coolant temperature to about 284.6 K (verify this by opening the model's coolant temperature scope), whereas the linear simulation predicts a reduction to 289.4 K. In other words, when plant moves in this direction, the linearized model's gain is too large. Still, the concentration goes to the setpoint rapidly.

Next, set the concentration setpoint to 8.07, i.e., a step-change of equal magnitude in the opposite direction. Run the simulation to obtain the concentration scope response shown below.



In this direction, the response is underdamped. You can verify that the controller changes the coolant temperature to 304.1 K, an increase of 6.0 K (recall the decrease of 13.5 K when the change was in the opposite direction). In other words, the controller's linear model underpredicts the effect of a coolant temperature increase. If the setpoint were reduced significantly, the closed-loop system would become unstable (try 7.0, for example).

Thus, the controller's effective operating range is limited. If you wanted to operate at a low concentration, you'd need to determine a linearized model at that condition and use it to design another controller.

---

**Note** This is typical of strongly nonlinear plants. If you needed to operate a conventional controller over such a wide range, you might consider gain scheduling. Equivalently, you could define predictive controllers for several operating points and switch from one to another depending on the measured concentration (see "Simulations Involving Nonlinear Plants" on page 4-8 for an example of this).

---

For the tested range, however, the oscillations die out quickly. You can verify that the controller responds equally well to small, sustained disturbances (i.e.,  $\pm 0.5$  in feed concentration or  $\pm 3$  in feed temperature).

### Modifying the Controller Using the Design Tool

Keeping the design tool open makes it easy to test controller modifications. Each time you run a Simulink simulation, the MPC Controller block automatically obtains the latest settings for its **MPC controller** parameter (**MPC1** in the above tests).

It's also easy to compare several controllers. Suppose you wanted to try a different control interval (the controller designed above uses a 1-second interval). Create a new controller by right-clicking on the design tool's **MPC1** controller node, and selecting **Copy Controller** in the menu. This creates a duplicate controller called **MPC1\_copy**. Rename it **MPC2**. Select **MPC2** in the tree, activate its **Model and Horizons** tab and set **Control interval** to 2.

Now return to the controller block mask (if necessary, double-click on the block to open the mask) and change **MPC controller** from MPC1 to MPC2. Run a simulation with a disturbance or setpoint change, and then display the **Coolant Temperature** scope. You should see that the controller is now making step-wise adjustments every 2 minutes. As would be expected, controller performance degrades with less frequent measurement feedback, but not much.

To restore the original controller, just change MPC2 to MPC1 in the block mask.

### Exiting the Design Tool

When you close the design tool, you'll be prompted to save any projects you've created or edited. For details, see "Saving a Project" on page 3-70.

Once the design tool has closed, the MPC Controller block mask expects its controller object to be in your workspace. Thus, you'll also be prompted to export such controller objects. You need to do this if you plan to run your Simulink model with the design tool closed.

## Saving Your Work

You'll usually want to save your design so you can reuse or revise it. You can save individual controllers or an entire project.

When you close the design tool, you'll be prompted to save new or modified designs. You can also save manually to preserve an intermediate state or guard against an unexpected shutdown.

This section covers the following topics:

- “Exporting a Controller” on page 3-69
- “Saving a Project” on page 3-70

### Exporting a Controller

To save a controller, *export* it to your MATLAB workspace or to a MAT-file. The former allows you to continue using the controller in command-line functions or a Simulink block. Your workspace disappears when you exit MATLAB, however. A MAT-file is permanent, and you can reload it when needed in a subsequent MATLAB session.

The following example assumes that the design tool is open and in the state described in the previous section. Suppose you want to export the controller to your workspace. Expand the tree if necessary, right-click on **MPC2**, and select **Export Controller** from the resulting menu. The dialog below appears.



The default behavior is to export the selected controller to the workspace. Click **Export** to confirm. You can verify the export by typing

whos

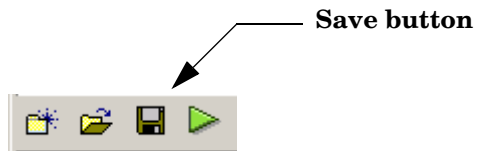
at the MATLAB prompt. The resulting list should include an mpc object named MPC2. Type

MPC2

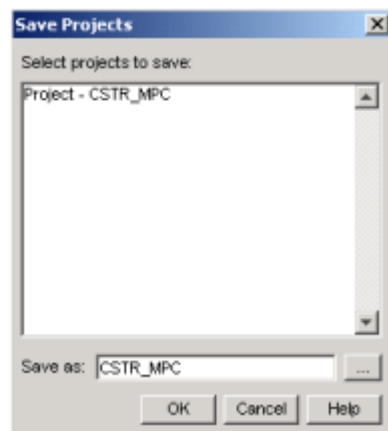
to display the object's properties.

## Saving a Project

To save your entire project, click the toolbar's Save button.



The following dialog appears.



**Figure 3-39: Dialog for Saving a Controller Design Project**

The default behavior saves the current project (named Project - CSTR\_MPC in this case) in a MAT-file (called CSTR\_MPC here). If the design tool had contained other projects, they would appear in the list, and you could select the ones you wanted to save.

The MAT-file will be saved in the default directory. To verify the location, click the browse (...) button and change the directory if necessary. When ready to save, click **OK**.

## Loading Your Saved Work

The following assumes that you've saved a project as described in the previous section. To reload this project, close the design tool if it's open. Also clear any mpc objects from your workspace. (Type `whos` at the MATLAB prompt for a list of objects.) For example, if MPC1 and MPC2 are in your workspace, type

```
clear MPC1 MPC2
```

to clear (remove) them.

If you've closed the CSTR\_MPC model, open it. (Figure 3-37 shows the model diagram). Double-click the MPC Controller block to open its mask, and verify that the **MPC Controller** parameter is set to MPC2.

---

**Note** If you were to attempt to run the CSTR\_MPC model at this stage, an error dialog would indicate that the MPC Controller block was unable to initialize. The MPC2 object specified in the block mask must be loaded into your workspace or be part of an active design tool task.

---

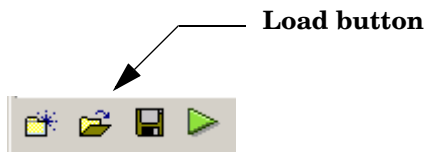
You could define the required MPC2 object in one of the following ways:

- Import MPC2 from a MAT-file (assuming you had saved it as explained in “Exporting a Controller” on page 3-69).
- Load the model's project file, which contains a copy of MPC2.

To use the second approach, open the design tool by typing

```
mpctool
```

in the Command Window. This creates a blank Model Predictive Control Toolbox project called MPCdesign. Click the Load button on the toolbar.





This opens a dialog similar to that shown in Figure 3-39. Use it to select the project file you've saved, and then click **OK** to load the project. It should appear in the tree. Verify that it contains a controller named MPC2.

Run the CSTR\_MPC model in Simulink. The block mask automatically retrieves **MPC2** from the design tool, and the simulation runs. In other words, loading the project automatically restores the link between the design tool and the MPC Controller block.



# Using Model Predictive Control Toolbox Functions

---

Controller Definition (p. 4-2)	Creates and modifies a controller using Model Predictive Control Toolbox functions
Linear Simulations (p. 4-6)	Simulates the closed-loop response of a linear system that includes a Model Predictive Control Toolbox controller
Simulations Involving Nonlinear Plants (p. 4-8)	Uses functions for flexible, closed-loop simulation of systems with nonlinear plants.
Analysis Tools (p. 4-14)	Analyzes Model Predictive Control Toolbox controllers

### Controller Definition

Chapter 3, “The Design Tool” showed how to use the Model Predictive Control Toolbox design tool to create a controller and test it. You might prefer to use functions instead. They allow access to options not available in the design tool, as well as automation of repetitive tasks using scripts and customized plotting.

This section covers the following topics:

- “Creating a Controller Object” on page 4-2
- “Viewing and Altering Controller Properties” on page 4-3

### Creating a Controller Object

The following uses the CSTR model described in Chapter 2, “Building Models” as an example. To follow along, verify that the model’s LTI object is in your MATLAB workspace (if necessary, create it as explained in “Chemical Reactor Example” on page 2-5, and set its label and signal type properties as explained in “LTI Properties for the CSTR Example” on page 2-7).

The `mpc` function is the starting point for controller design. For example, type

```
Ts = 1;  
MPCobj = mpc(CSTR, Ts);
```

to create a controller based on the CSTR model with a control interval of 1 time unit, and all other parameters at their default values.

---

**Note** MPCobj is an *mpc object*. It contains a complete controller as defined by the Model Predictive Control Toolbox.

---

To display the controller’s properties, type

```
display(MPCobj)
```

or type the object’s name without a trailing semicolon.

## Viewing and Altering Controller Properties

Once you've defined an mpc object, it's easy to alter its properties. For a description of the editable properties, type

```
mpcprops
```

To display the list of properties and their current values, type

```
get(MPCobj)
```

which in this case displays

```
ManipulatedVariables (MV): [1x1 struct]
  OutputVariables (OV): [1x2 struct]
  DisturbanceVariables (DV): [1x1 struct]
    Weights (W): [1x1 struct]
      Model: [1x1 struct]
        Ts: 1
    Optimizer: [1x1 struct]
  PredictionHorizon (P): 10
  ControlHorizon: 2
  History: [2e+003 7 21 20 18 20.1]
  Notes: {}
  UserData: []
```

To alter one of these properties, you can use the syntax

```
ObjName.PropName = value;
```

where ObjName is the object name, and PropName is the property you want to set. For example, to change the prediction horizon from 10 (the default) to 15, type

```
MPCobj.P = 15;
```

---

**Note** You can abbreviate property names provided that the abbreviation is unambiguous.

---

As shown above, many of the properties are MATLAB *structures* containing additional properties. For example, type

```
MPCobj.MV
```

which displays

```
Min: -Inf
Max: Inf
MinECR: 0
MaxECR: 0
RateMin: -Inf
RateMax: Inf
RateMinECR: 0
RateMaxECR: 0
Target: 'nominal'
Name: 'T_c'
Units: ''
```

This shows that the default controller has no constraints on the manipulated variable. To include constraints as shown in Figure 3-18, you could type

```
MPCobj.MV.Min = -10;
MPCobj.MV.Max = 10;
MPCobj.MV.RateMin = -4;
MPCobj.MV.RateMax = 4;
MPCobj.MV.Units = 'Deg C';
```

or more compactly using the set syntax

```
set(MPCobj, 'MV', struct('Min', -10, 'Max', 10, ...
    'RateMin', -4, 'RateMax', 4, 'Units', 'Deg C'));
```

---

**Note** The Units property is for display purposes only and is optional.

---

There are two outputs in this case, so `MPCobj.OV` is a 1-by-2 structure. To set measurement units to the values shown in Figure 3-12, you could type

```
MPCobj.OV(1).Units = 'Deg C';  
MPCobj.OV(2).Units = 'kmol/m^3';
```

Finally, check the default weights by typing

```
MPCobj.W
```

which displays

```
ManipulatedVariables: 0  
ManipulatedVariablesRate: 0.1000  
OutputVariables: [1 1]  
ECR: 100000
```

Change to the values shown in Figure 3-12 by typing

```
MPCobj.W.ManipulatedVariablesrate = 0.3;  
MPCobj.W.OutputVariables = [1 0];
```

When setting the properties manually you can specify a different weight for each step in the prediction horizon (this isn't possible when using the design tool). For example,

```
MPCobj.W.OutputVariables = [0.1 0; 0.2 0; 0.5 0; 1 0];
```

de-emphasizes setpoint tracking errors early in the prediction horizon. The default weight of 1 would be used for the fourth step and beyond. Constraints can also be time varying. Type `mpcprops` for details.

You could also alter the controller's disturbance rejection characteristics using functions that parallel the design tool's disturbance modeling options (described in "Disturbance Modeling and Estimation" on page 3-34). See the reference pages for the `setestim`, `setindist`, and `setoutdist` functions.

### Linear Simulations

The Model Predictive Control Toolbox functions allow you to perform linear closed-loop and open-loop simulations. This section covers the following topics:

- “Using the `sim` Function” on page 4-6
- “Saving Calculated Results” on page 4-6
- “Simulation Options” on page 4-7

#### Using the `sim` Function

To run a linear simulation, use the `sim` function. For example, given the `MPCobj` controller defined in the previous section, type

```
T = 26;  
r = [2 0];  
sim(MPCobj, T, r);
```

This simulates the closed-loop response for a duration of 26 control intervals with a setpoint of 2 for the first output (the reactor temperature) and 0 for the second output (the residual concentration). Recall that the second output’s tuning weight is zero (see the discussion in “Output Weights” on page 3-24), so its setpoint is ignored.

By default, the same linear model is used for controller predictions and the plant, i.e., there is no plant/model mismatch. You can alter this as shown in “Simulation Options” on page 4-7.

When you use the above `sim` syntax (no output variables), it automatically plots the plant inputs and outputs (not shown, but see Figure 3-13 and Figure 3-14 for results of a similar scenario).

#### Saving Calculated Results

If you’d like to save simulation results in your workspace, use the following `sim` function format:

```
[y, t, u] = sim(MPCobj, T, r);
```

This suppresses automatic plotting, instead creating variables `y`, `t`, and `u`, which hold the computed outputs, time, and inputs, respectively. A typical use



is to create customized plots. For example, to plot both outputs on the same axis versus time, you could type

```
plot(t, y)
```

## Simulation Options

You can modify simulation options using the `mpcsimopt` function. For example, the code

```
MPCOpts = mpcsimopt;  
MPCOpts.Constraints = 'off';  
sim(MPCObj, T, r, MPCOpts)
```

runs an unconstrained simulation. Comparing to the case described in “Using the `sim` Function” on page 4-6, the controller’s first move is now exceeds 4 units (the specified rate constraint).

Other options include the addition of a specified noise sequence to the manipulated variables or measured outputs, open-loop simulations, a look-ahead option for better setpoint tracking or measured disturbance rejection, and plant/model mismatch.

For example, the following code defines a new plant model having gains 50% larger than those in the CSTR model used in the controller, then repeats the above simulation:

```
Plant = 1.5*CSTR;  
MPCOpts.Model = Plant;  
sim(MPCObj, T, r, MPCOpts)
```

In this case, the plant/model mismatch degrades controller performance, but only slightly. Degradation can be more severe and must be tested on a case-by-case basis.

# Simulations Involving Nonlinear Plants

The `sim` function accepts linear plant models only. If your plant is a nonlinear Simulink model, you could control it as demonstrated in “Nonlinear Plants” on page 3-57. To do so, you would define the controller using the `mpc` function, as illustrated in “Creating a Controller Object” on page 4-2, and then use it as the MPC Controller block mask’s **MPC Controller** parameter. You would then be ready to run the Simulink model closed-loop.

---

**Note** You would also need to define a linear model to be used with the `mpc` function. Linearization is the usual approach. See “Linearization Using Simulink Control Design” on page 2-19.

---

Alternatively, your nonlinear model might be a MEX-file, or you might want to include features unavailable in the MPC Controller block, such as a custom state estimator. The `mpcmove` function is the Model Predictive Control Toolbox computational engine, and you can use it in such cases. The disadvantage is that you must duplicate the infrastructure that the `sim` function and the MPC Controller block provide automatically.

The rest of this section covers the following topics:

- “A Nonlinear CSTR Application” on page 4-8
- “Example Code for Successive Linearization” on page 4-9
- “CSTR Results and Discussion” on page 4-10

## A Nonlinear CSTR Application

The CSTR model described in “Using Simulink to Develop LTI Models” on page 2-19 is a strongly nonlinear system. As shown in “Nonlinear Plants” on page 3-57, a controller can regulate this plant, but degrades (and might even become unstable) if the operating point changes significantly.

The objective of this example is to redefine the predictive controller at the beginning of each control interval so that its predictive model, though linear, represents the latest plant conditions as accurately as possible. This will be done by linearizing the nonlinear model repeatedly, allowing the controller to adapt as plant conditions change. See references [1] and [2] for more details on this approach.

## Example Code for Successive Linearization

In the following code, the simulation begins at the CSTR model's nominal operating point (concentration = 8.57) and moves to a low concentration (= 2) where the reaction rate is much higher. The required code is as follows:

```
[sys, xp] = CSTR_INOUT([],[],[], 'sizes');
up = [10 298.15 298.15];
u = up(3);
tsave = []; usave = []; ysave = []; rsave = [];
Ts = 1;
t = 0;
while t < 40
    yp = xp;
    % Linearize the plant model at the current conditions
    [a,b,c,d]=linmod('CSTR_INOUT', xp, up );
    Plant = ss(a,b,c,d);
    Plant.InputGroup.ManipulatedVariables = 3;
    Plant.InputGroup.UnmeasuredDisturbances = [1 2];
    Model.Plant = Plant;
    % Set nominal conditions to the latest values
    Model.Nominal.U = [0 0 u];
    Model.Nominal.X = xp;
    Model.Nominal.Y = yp;
    dt = 0.001;
    Options = simset('InitialState', xp);
    [T, XP, YP] = sim('CSTR_INOUT', [t t+dt], Options, ...
        [t up; t+dt up]);
    Model.Nominal.DX = (1/dt)*(XP(end,:) - xp(:));
    % Define the MPC Toolbox controller for the latest model
    MPCObj = mpc(Model, Ts);
    MPCObj.W.Output = [0 1];
    % Ramp the setpoint
    r = max([8.57 - 0.25*t, 2]);
    % Compute the control action
    if t <= 0
        xd = [0; 0];
        x = mpcstate(MPCObj, xp, xd, [], u);
    end
    u = mpcmove(MPCObj, x, yp, [0 r], []);
    % Simulate the plant for one control interval
```

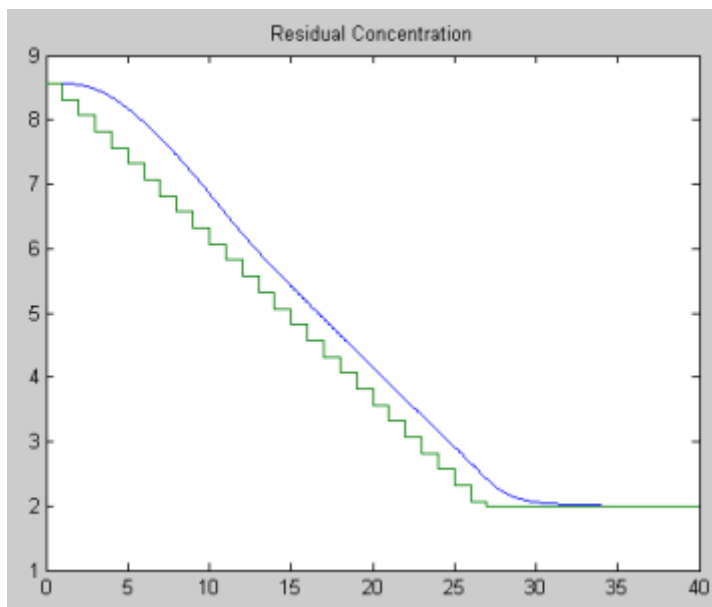
```
up(3) = u;
Options = simset('InitialState', xp);
[T, XP, YP] = sim('CSTR_INOUT', [t t+Ts], Options, ...
    [t up; t+Ts up]);
% Save results for plotting
tsave = [tsave; T];
ysave = [ysave; YP];
usave = [usave; up(ones(length(T),1),:)]);
rsave = [rsave; r(ones(length(T),1),:)]);
xp = XP(end,:);
t = t + Ts;
end
figure(1)
plot(tsave,[ysave(:,2) rsave])
title('Residual Concentration')
figure(2)
plot(tsave,usave(:,3));
title('Coolant Temperature')
```

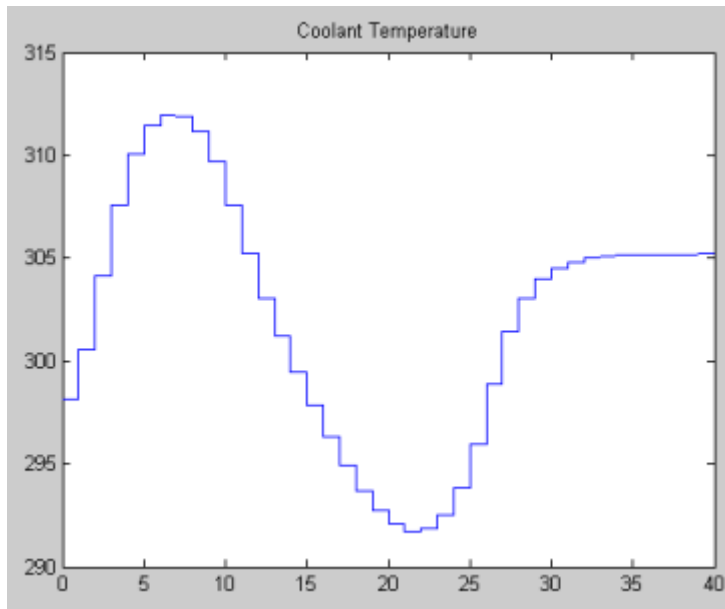
### CSTR Results and Discussion

The plotted results appear below. Note the following points:

- The setpoint is being ramped from the initial concentration to the desired final value (see the step-wise changes in the reactor concentration plot below). The reactor concentration tracks this ramp smoothly with some delay (see the smooth curve), and settles at the final state with negligible overshoot. The controller works equally well (and achieves the final concentration more rapidly) for a step-wise setpoint change, but it makes radical changes in coolant temperature (not shown) that would probably be difficult to achieve in practice.
- The final steady state requires a coolant temperature of 305.20 K (see the coolant temperature plot below). An interesting feature of this nonlinear plant is that if one starts at the initial steady state (coolant temperature = 298.15 K), stepping the coolant temperature to 305.20 and holding will not achieve the desired final concentration of 2. In fact, under this simple strategy the reactor concentration stabilizes at a final value of 7.88, far from the desired value! A successful controller must increase the reactor temperature until the reaction “takes off,” after which it must reduce the

coolant temperature to handle the increased heat load. The relinearization approach provides such a controller (see following plots).





- Function `linmod` relinearizes the plant as its state evolves. This function was discussed previously in “Linearization Using Simulink Functions” on page 2-24. The code also resets the model’s nominal conditions to the latest values.
- Function `mpc` defines a new controller based on the relinearized plant model. The output weight tuning ignores the temperature measurement, focusing only on the concentration.
- At  $t = 0$ , the `mpcstate` function initializes the controller’s extended state vector,  $x$ , which is an *mpcstate object*. Thereafter, the `mpcmove` function updates it automatically using the controller’s default state estimator. It would also be possible to use an Extended Kalman Filter (EKF) as described in [1] and [2], in which case the EKF would reset the `mpcstate` input variables at each step.
- The `mpcmove` function uses the latest controller definition and state, the measured plant outputs, and the setpoints to calculate the new coolant temperature at each step.

- The Simulink `sim` function simulates the nonlinear plant from the beginning to the end of the control interval. Note that the final condition from the previous step is being used as the initial plant state, and that the plant inputs are being held constant during each interval.

Remember that a conventional feedback controller or a fixed Model Predictive Control Toolbox controller tuned to operate at the initial condition would become unstable as the plant moves to the final condition. Periodic model updating overcomes this problem automatically and provides excellent control under all conditions.

### Analysis Tools

There are many ways to analyze a controller design. This section highlights two functions that support analysis of Model Predictive Control Toolbox controllers:

- “Steady-State Gain Computation” on page 4-14
- “Controller Extraction” on page 4-14

#### Steady-State Gain Computation

The `cloffset` function computes the closed-loop, steady-state gain for each output when subjected to a sustained, 1-unit disturbance in each output. It assumes that no constraints will be encountered.

For example, consider the controller operating at the final steady-state of the nonlinear CSTR of the previous section. To compute its gain, type

```
cloffset(MPCobj)
```

which gives the result

```
ans =
```

```
    1.0000    14.5910  
   -0.0000    -0.0000
```

The interpretation is that the controller doesn’t react to a sustained disturbance of 1 unit in the first output (the reactor temperature). Recall that this output has zero weight, so the controller ignores it. The same disturbance has no effect on the second output (the 2,1 element is zero).

If there is a 1-unit disturbance in the second output, the controller reacts, and the first output increases 14.59 units. The controller keeps the second output at its setpoint (the 2,2 element is zero).

#### Controller Extraction

Use the `ss` function to obtain an LTI representation of an unconstrained Model Predictive Control Toolbox controller. You can then use this to analyze closed-loop frequency response, etc.



---

For example, consider the controller designed in “Creating a Controller Object” on page 4-2. To extract the controller, you could type

```
MPCss = ss(MPCobj);
```

You could then construct an LTI model of the closed-loop system using the feedback function (see the Control Systems Toolbox for details) by typing

```
CSTRd = c2d(CSTR, MPCss.Ts);  
CLsys = feedback(CSTRd, MPCss, 1, 2);
```

---

**Note** The CSTR model must be converted to discrete form with the same control interval as the controller.

---

You could then type

```
eig(CLsys)
```

to verify that all closed-loop poles are within the unit circle, or

```
bode(CLsys)
```

to compute a closed-loop bode plot, etc.

### Bibliography

- [1] Lee, J.H. and N.L. Ricker, "Extended Kalman Filter Based Nonlinear Model Predictive Control," *Ind. Eng. Chem. Res.*, Vol. 33, No. 6, pp. 1530-1541(1994).
- [2] Ricker, N. L., and J. H. Lee "Nonlinear Model Predictive Control of the Tennessee Eastman Challenge Process," *Computers & Chemical Engineering*, Vol. 19, No. 9, pp. 961-981(1995).

## B

blocking 3-27

## C

cloffset function 4-14

closed-loop

    design tool simulations 3-16

constraints

    input 3-30

    manipulated variable 3-30

    output 3-42

    softening 3-42

Control and Estimation Tools Manager 3-2

control horizon

    specification 3-20

control interval

    specification 3-20

controller settings

    blocking 3-27

    control interval 3-20

    delay compensation 3-50

    disturbance estimation 3-34

    get function 4-3

    horizons 3-20

    input constraints 3-30

    input weights 3-26

    move suppression 3-25

    output weights 3-24

    plant model 3-20

    prediction horizon 3-20

    rate weights 3-25

    set function 4-3

    weights 3-22

controllers

    comparisons 3-27

    creation 4-2

    design tool 3-20

    design tool list 3-9

    exporting 3-69

    LTI object extraction 4-14

    mpc function 4-2

    property settings 4-3

    robustness 3-46

CSTR (Continuous Stirred-Tank Reactor)

    linear model 2-5

    nonlinear model 2-20

## D

delays

    controller settings for 3-50

description

    specification 3-6

design tool

    loading models 3-3

    starting 3-2

    tree navigation 3-7

*See also* views

disturbance estimation 3-34

disturbances

    specification in simulations 3-16

documentation 1-4

## E

estimation

    disturbances 3-34

## G

gain

    closed-loop 4-14

**H**

horizons  
specification 3-20

**I**

import  
plant model 3-4

importing  
controller 3-72

InputGroup property 2-7

InputName property 2-8

inputs  
manipulated variables 2-3  
measured disturbances 2-3  
names 2-8  
noise 2-16  
nominal value 3-6  
type specification 2-9  
types 2-3  
unmeasured disturbances 2-3

**L**

linearization  
operating point 2-21  
repeated 4-9  
Simulink Control Design 2-19  
Simulink functions 2-24

linmod function 2-24

loading  
controllers 3-72  
project 3-72

LTI objects  
properties 2-7

**M**

manipulated variables  
definition 2-3

measured disturbances  
definition 2-3

measured outputs  
definition 2-3

Model Predictive Control (MPC)  
related products 1-5  
Toolbox 1-2

models  
characteristics 2-13  
delays 3-49  
design tool list 3-8  
disturbances 3-34  
linearization 2-19  
loading into design tool 3-3  
LTI 2-4  
LTI chemical reactor 2-5  
noise inputs 2-16  
nonlinear 2-19  
nonlinear CSTR example 3-57  
nonlinear simulation 4-8  
nonsquare 3-55  
perturbation 3-46  
plant 2-2  
prediction error 3-46  
relinearizing 3-63  
Simulink 2-19  
state-space 2-5  
step response 2-17  
system identification 2-14  
transfer function 2-4  
zero/pole/gain 2-5  
*See also* system identification

MPC (Model Predictive Control)

*See* Model Predictive Control

mpc objects

- conversion to state-space 4-14
- creating 4-2

mpcmove function 4-9

mpcstate objects 4-10

mpctool function

- starting 3-2

MV (manipulated variables)

- See* manipulated variables

## N

names

- specification using design tool 3-6

nominal values

- specification 3-6

nonlinear plants 3-57

nonsquare plants 3-55

## O

objects

- LTI 2-7
- mpc 4-2
- mpc conversion to ss 4-14
- mpcstate 4-10

open-loop

- design tool simulations 3-16

operating point

- new 2-22
- specification 2-21

OutputGroup property 2-7

OutputName property 2-8

outputs

- measured 2-3
- names 2-8

- nominal value 3-6
- setpoint definition 3-12
- type specification 2-9
- types 2-3
- unmeasured 2-3

## P

plant/model mismatch 3-46

plants

- input/output signals 2-3
- MIMO 2-11
- nonlinear 3-57
- nonlinear simulation 4-8
- nonsquare 3-55

properties

- mpc object 4-3

## R

response plots 3-14

- data markers 3-14

robustness

- controller prediction error 3-46
- testing 3-46

## S

saving

- controllers 3-69
- projects 3-70

scenarios

- design tool list 3-10
- specification 3-12

setpoints

- specification 3-12

signal labels

- description 3-6
- name 3-6
- units 3-6
- signals
  - inputs 2-3
  - outputs 2-3
  - types 2-3
- sim function
  - options 4-7
  - simulations 4-6
- simulations
  - closed-loop 3-16
  - conditions 3-12
  - linear 3-12
  - nonlinear 3-66
  - open loop 3-16
  - options 4-7
  - sim function 4-6
  - starting in design tool 3-14
  - toolbar icon 3-13
  - See also* scenarios
- soft constraints 3-42
- ss function 2-6
- state estimation 3-34
- SysID (System Identification)
  - See* system identification
- system identification
  - example 2-14
  - model conversion 2-15
  - noise inputs 2-16
  - step response 2-17

**T**

- tf function 2-4
- transfer functions
  - specification 2-4

- tree
  - navigation 3-7
- types
  - specification using design tool 3-6

**U**

- units
  - specification 3-6
- unmeasured outputs
  - definition 2-3

**V**

- views
  - constraint softening 3-43
  - controller settings 3-20
  - controllers 3-9
  - plant models 3-8
  - scenarios 3-10
  - signal properties 3-7
  - See also* design tool

**W**

- weights
  - inputs 3-26
  - move suppression 3-25
  - output 3-24
  - rate 3-25
  - setpoint tracking 3-24
  - tuning 3-22

**Z**

- zpk function 2-5